

# Performance Enhancement of Inverted Pendulum System by Using Type 2 Fuzzy Sliding Mode Control (T2FSMC)

Mardlijah<sup>\*\*1</sup>, Muh.Abdillah<sup>#2</sup>, Ach Jazidie<sup>#3</sup>, Basuki Widodo<sup>\*4</sup>, Ari Santoso<sup>#5</sup>

<sup>#</sup>*Electrical- Department, ITS  
Kampus ITS Sukolilo, Surabaya Indonesia*

<sup>1</sup>*mardlijah@matematika.its.ac.id*

<sup>2</sup>*abdillah@elect-eng.its.ac.id*

<sup>3</sup>*santoso@elect-eng.its.ac.id*

<sup>5</sup>*jazidie@ee.its.ac.id*

<sup>\*</sup>*Mathematics Department, ITS  
Kampus ITS Sukolilo, Surabaya Indonesia*

<sup>4</sup>*b\_widodo@matematika.its.ac.id*

**Abstract—** The drawbacks of using FSMC when controlling the position motion on a non linear inverted pendulum system are the chattering problem and the robustness against parameter uncertainty and disturbances. This paper consider about the implementation of Fuzzy Type 2 in combining with SMC (T2FSMC) to enhance the ability of FSMC to overcome both of those problems. To be more optimally damp the oscillation, the range of fuzzy membership is determined based on the minimum ITAE (integral of time absolute error) value while the rule is constructed based on the product between the sliding surfaces and its derivatives. To enhance robustness performance against disturbances and parameter uncertainties, the range of non crisp membership value (footprint uncertainty) of Fuzzy Type 2 is determined by considering the minimum sum of ITAE values at several conditions with different plant parameter. It is proved from the simulation that the proposed method can solve chattering problem more effectively and also more robust against disturbances and parameter uncertainties than T1FSMC.

**Keywords—** Fuzzy Sliding Mode Control, Type 2 Fuzzy Sliding Mode Control, Inverted Pendulum.

## I. INTRODUCTION

The control of non linear system becomes an important topic as almost all phenomena of real world is a non linear system. One of the nonlinear phenomena frequently studied is inverted pendulum e. g a benchmark of non linear systems. There are many methods have been proposed to control this non linear system. One of them is Sliding Mode Control (SMC). SMC is a robust nonlinear discontinuous feedback control technique which is addressed to uncertainty and dynamics of plant.[1,2]. The main drawback of SMC is chattering. Many researches have been proposed to overcome

this, such as introducing a small boundary layer around sliding surface for suppressing the chattering in order to achieve a better accuracy [3]. Sometimes, trajectory is not longer converges to sliding surface so that SMC needs the general structure of the plant and parameter uncertainties to remain within known intervals. In the recent years, some techniques have been emerged for controlling such problem, especially techniques base on fuzzy logic systems (FLS) [4, 5]. Fuzzy logic theory provides an important tool for utilization of human expert knowledge in complement to mathematical knowledge. This is mainly due to the possibility of making use of fuzzy knowledge-based control to deal with systems which have dynamics are not so well understood and which have models can not be so accurately established [4] Many recent researches have utilized a fuzzy sliding mode control approach (FSMC) i.e. type-1 FLS- to handle the chattering, uncertainties and disturbances. Unfortunately, the information that is used to construct the rules in a FLS is quite deterministic. This causes Type-1 FLS are unable to directly handle towards the rule of uncertainties, making they are so sensitive against disturbance or quickly changing phenomenon [6, 8]. Anticipating this problem, in this paper we use an interval type-2 fuzzy sliding mode control combined with SMC (T2FSMC) for a class of uncertain SISO nonlinear pendulum systems. Type-2 FLS involved in this paper has antecedent or consequent membership functions, hoping it can handle the rule of uncertainties caused by disturbances. Here, in controlling the inverted pendulum, we will also compare the result of the proposed method (T2FSMC) with FSMC.

## II. METHODOLOGY

### A. Developing FSMC Method

Before developing the proposed method, for comparison, we firstly developed Fuzzy Sliding Mode Control Methods

which is contains of two main components, that are SMC part and FLC part, as seen at the following block diagram:

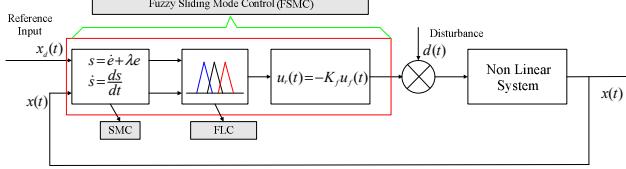


Figure 1 Block Diagram of FSMC

It is used as inputs the reference variable  $x_d(t)$  and output feedback  $x(t)$  resulting an error  $e$ . Those are used by SMC to compute the sliding surface  $s$  and its gradient  $\dot{s}$ . In spite of using analityc SMC formulae, we adopt Mamdani type fuzzy reaching control, as which is used by Zhu et.al [5] in computing the following control signal:

$$u = -K_f u_f(t) \quad (1)$$

Where  $K_f$  is a normalization factors. The fuzzy output variable,  $u_f$  is continuously adjusted using an if-then rule-base with respect to both  $s$  and  $\dot{s}$ . By using the expertise in design of sliding-mode controllers, a big switching gain will force the state trajectories to approach the sliding surface ( $s=0$ ) rapidly; but unfortunately at the same time, the chattering is excited. When a tracking error increase then the value of  $s$  also will increase ; therefore, the switching gain should be correspondingly increased and vice versa. Furthermore, when the state trajectories deviate from sliding surface,  $s^T \dot{s} > 0$  the switching gain should be increased if the value of  $\dot{s}$  large, this is taken to force the trajectories back, and vice versa. When the state trajectories are approaching the sliding surface,  $s^T \dot{s} < 0$ , if the value of  $\dot{s}$  is large the switching gain should be decrease to reduce chattering, and vice versa. In executing the above rules, the stability will be guaranteed if the reaching condition  $\dot{V} = \frac{1}{2} \frac{d}{dt}(S^2) \leq -\eta |S|$  which is derived from Lyapunov Stability Criterion is fulfilled [10]. Implementation such kind of fuzzy rule in combination with SMC on inverted pendulum plant is success to reduce the chattering problem [5]. In this research we will make perfect this method by replacing Fuzzy Type 1 to be type 2 with expectations of better performance on reducing chattering and time convergence.

### B. Developing T2FSMC Method

As discuss before, drawback of T1FSMC is the lack of chattering robustness against external disturbances. In our research, we propose the utilization of Fuzzy type 2 in combining with SMC in order to achieve smoother chattering and faster convergence. The block diagram of the T2FSMC is depicted as follows.

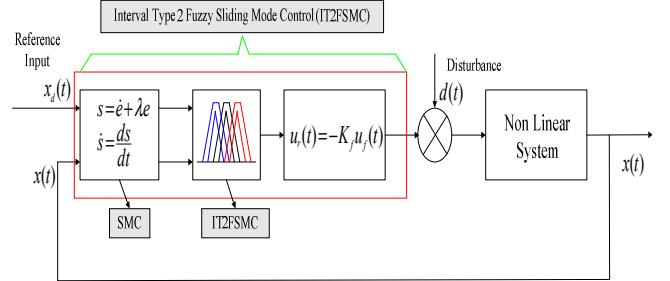


Figure 2 Block diagram of IT2FSMC

As seen at Fig. 2, the control system is still the same as at Fig 1 except we replace the Fuzzy Type 1 to be The Type 2. The advantage of Fuzzy type 2 lays on its triangle membership value expressed in a range. These ranges are very suitable in dealing with the values uncertainties of  $s$  and  $\dot{s}$  caused by the plant parameter uncertainties and disturbances influences. The number of rules can be reduced in comparison with type 1. A general T2 FLS is depicted at Fig. 3 [6, 7].

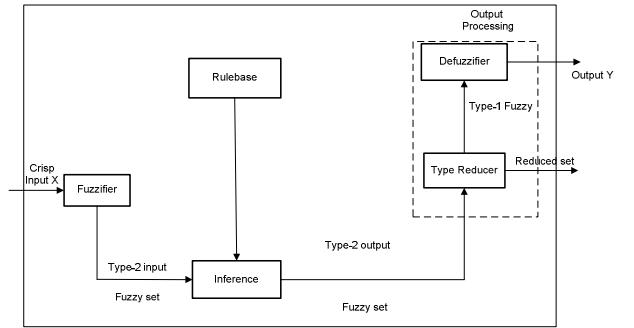


Figure 3 Block diagram of IT2FLC

It is similar to T1 FLS except the defuzzifier block of a T1 FLS is replaced by the output processing block in a T2 FLS. That block consists of type-reduction (TR) followed by defuzzification. To guarantee the robustness against parameter uncertainties and disturbances, the concept of smallest ITAE sum value is used to determine the fuzzy membership and its footprint uncertainty as shown at Fig.4. Eah ITAE value is computed at nominal operating condition and several representative boundary conditions.

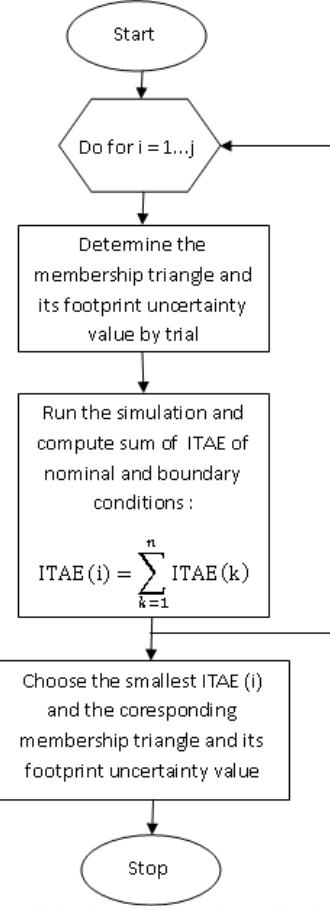


Fig.4 Determination of the fuzzy membership and its footprint uncertainty

Global explanation about type2 is as follows:

Fuzzifier maps a crisp point  $x = (x_1, x_2, x_3 \dots)^T \in X$  into type 2 fuzzy set  $\tilde{A}_x$  in  $\mathbf{X}$ , interval type-2 fuzzy sets.

$\tilde{A}_x$  is a type-2 fuzzy singleton if  $\mu_{\tilde{A}_x}(X) = 1/1$  for  $x = x^1$  and  $\mu_{\tilde{A}_x}(X) = 1/0$  for  $x \neq x^1$ .

Rules in type 1 and type 2 are same except the antecedents and the consequents will be represented by type-2 fuzzy sets. Then for type-2 FLS with  $p$  inputs  $x_1, x_2, x_3 \dots x_p \square X$  and one output  $y \in Y$  (Multiple Input Single Output (MISO)), if we assume there are  $M$  rules, the  $l$ th rule in the type-2 FLS can be written as follows:

$$R^l : \text{IF } x_1 \text{ is } F_1^l \text{ and } \dots \text{ and } x_p \text{ is } F_p^l \text{ THEN } y \text{ is } G^l, \\ l = 1, 2, \dots, M$$

The inference engine combines rules and gives a mapping from input type-2 fuzzy sets to output type-2 fuzzy sets. It is need to compute the join  $\sqcup$  (unions) and the meet  $\Pi$  (intersections) of type-2 relations. If  $F_1^l \times \dots \times F_p^l = \tilde{A}^l$  then  $R^l : F_1^l \times \dots \times F_p^l \rightarrow G^l = \tilde{A}^l \rightarrow G^l, l = 1, 2, \dots, M$ ,

$R^l$  is described by membership function

$$\mu_{R^l}(x, y) = \mu_{R^l}(x_1, \dots, x_p, y) = \mu_{\tilde{A}^l \rightarrow G^l}(x, y) = \\ [\prod_{i=1}^p \mu_{F_i^l}(x_i)] \prod \mu_{G^l}(y),$$

Type reducer will generate type 1 fuzzy set output, and then is converted into a crisp output through the fuzzifire. Here center of sets (cos) type reduction  $Y_{cos}$  is used, expressed as

$$Y_{cos}(\mathbf{x}) = [y_l, y_r] = \\ \int_{y^1 \in [y_l^1, y_r^1]} \dots \int_{y^M \in [y_l^M, y_r^M]} \int_{f^1 \in [f_l^1, f_r^1]} \dots \int_{f^M \in [f_l^M, f_r^M]} 1 / \frac{\sum_{i=1}^M f^i y^i}{\sum_{i=1}^M f^i}$$

$y_l$  and  $y_r$  correspond to the centroid of type2 interval consequent set  $\tilde{G}^l$ ,

$$C_{\tilde{G}^l} = \int_{\theta_1 \in J_{y^1}} \dots \int_{\theta_N \in J_{y^N}} 1 / \frac{\sum_{i=1}^N y_i \theta_i}{\sum_{i=1}^N \theta_i} = [y_l^l, y_r^l]$$

If the values of  $f_i$  and  $y_i$  that associated with  $y_l$  are denoted as  $f_l^l$  and  $y_l^l$ , and either for  $y_r$  then

$$y_l = \frac{\sum_{i=1}^M f_l^i y_l^i}{\sum_{i=1}^M f_l^i}$$

and

$$y_r = \frac{\sum_{i=1}^M f_r^i y_r^i}{\sum_{i=1}^M f_r^i}$$

After an interval  $Y_{cos}$  is obtained in the type reducer, the average of  $y_l$  and  $y_r$  is use to defuzzify that, we have

$$y(\mathbf{x}) = \frac{y_l + y_r}{2}$$

Figure 5 - 7 show that to design triangle membership, which the number and the uncertainty range of the membership function are determined base on the best damping performance of the controlled signals. Table I shows the resulted rule-base, which is determined based on the multiplication value between surface transpose and surface derivative,  $s^T \dot{s}$ .

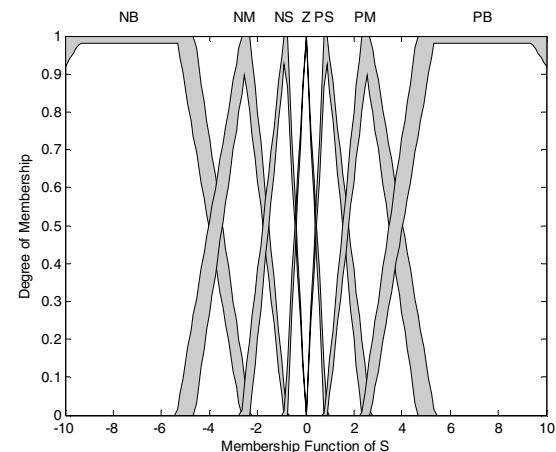


Figure 5 Membership Function of S

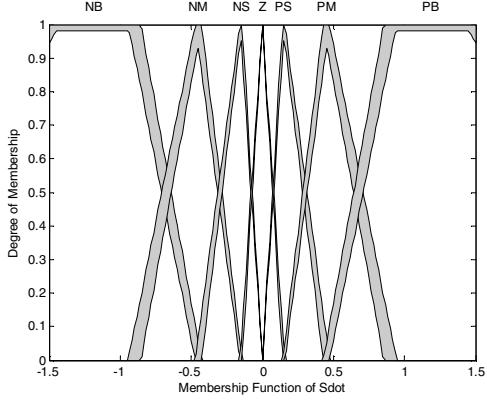


Figure 6 Membership Function of  $\dot{S}$

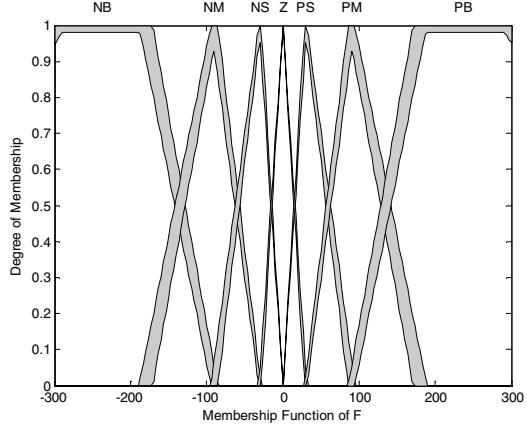


Figure 7 Membership Function of  $u$

TABLE I  
RULE - BASE OF TYPE2FSMC

		$\dot{S}$						
		NB	N M	NS	Z	PS	PM	PB
S	PB	Z	NS	NS	N M	N M	NB	NB
	PM	PS	Z	NS	NS	N M	N M	NB
	PS	PS	PS	Z	NS	NS	N M	N M
	Z	PM	PS	PS	Z	NS	NS	N M
	NS	PM	PM	PS	PS	Z	NS	NS
	NM	PB	PM	PM	PS	PS	Z	NS
	NB	PB	PB	PM	PM	PS	PS	Z

### C. Mathematical Modelling of The Inverted Pendulum on Cart

The plant chosen in implementing of the IT2FSMC is inverted pendulum, as can be seen at Figure 8. It consists of

one DC motor, end rail switch, encoder, cart and two poles. The mathematical model of this plant is as follows:

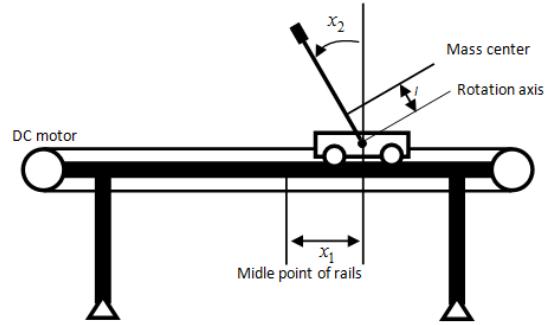


Figure 8 Inverted Pendulum on Cart

$$x_1' = x_3 ,$$

$$x_2' = x_4 ,$$

$$\begin{aligned} x_3' &= \frac{a(F - T_c - \mu \cdot x_4^2 \sin x_2) + l \cos x_2 (\mu \cdot g \sin x_2 - f_p x_4)}{J + \mu \cdot l \sin^2 x_2} \\ x_4' &= \frac{l \cos x_2 (F - T_c - \mu \cdot x_4^2 \sin x_2) + \mu \cdot g \sin x_2 - f_p x_4}{J + \mu \cdot l \sin^2 x_2} \end{aligned} \quad (2)$$

and

$$a = l^2 + \frac{J}{m_c + m_p}; \quad \mu = (m_c + m_p)l \quad (3)$$

$x_1$  = cart position,

$x_2$  = angle of pendulum,

$x_3$  = cart velocity, and

$x_4$  = angular velocity of pendulum.

The parameter of inverted Pendulum which we use, can be seen on Tabel II.

TABLE III  
PARAMETER OF INVERTED PENDULUM

Parameter	Symbols	Value
Cart mass [kg]	$m_c$	1.120
Pendulum mass [kg]	$m_{pw}$	0.095
Pendulum poles mass[kg]	$m_{ps}$	0.025
Rails distance[m]	$R_l$	1.000
Length of poles [m]	$l_p$	0.402
Distance of center of mass of poles [m]	$l_{po}$	0.146
Distance of pendulum mass [m]	$l_c$	0.041
Distance of center of mass of pendulum [m]	$l_{co}$	0.347
Radius of pendulum [m]	$r_c$	0.02
Radius of poles [m]	$r_p$	0.006
Static friction force [N]	$F_s$	2.2813
Coulumbic friction force[N]	$F_c$	2.5316
Maximum control force [N]	$M$	17.46
Minimum control force [N]	$DZu$	1.379
Maximum cart velocity [N]	$DZcv$	0.008
Minimum swing velocity of pendulum	$DZpv$	3.374

[rad/sec]		
Pendulum friction [ $\text{kg}^*\text{m}^2/\text{sec}$ ]	$J_p$	0.0139
Inertia moment of pendulum [ $\text{kg}^*\text{m}^2$ ]	$J$	0.0135
Maximum control voltage [V]	$V_m$	2.5

Our goal is to control the angle  $x_2$  by regulating DC Motor Voltage via the force (control signal). The detail relationship between control signal and DC Motor Voltage has been derived as seen at ref [9].

### III. SIMULATION AND RESULT ANALYSIS

The above model together with other blocks of SMC and Type2 Fuzzy will be simulated using Matlab Simulink. The triggering signal is given at reference input point as step signal. It is desired that the IT2FSMC control method can give optimal damping so that the oscillation of position will not take a long time to converge and robust against disturbances. The impulse signals are taken as disturbance and it is substituted into the system together with the control signal. The result of the proposed method will be compared with the result of FSMC method.

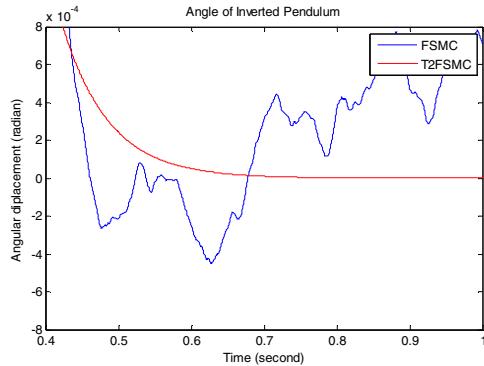


Figure 9 Angle of Inverted Pendulum

Figure 9 shows the result of the angle of the inverted pendulum as a response against impulse trigger signal. It is clearly shown that the angle response, which is resulted using the proposed method (T2FSMC), is much more stable compared with one resulted from FSMC. In line with this result, it is a response of control signal (force) and angular speed (figure 10 and figure 11). All of these explain a fact that the representation of triangle membership in a range in Fuzzy Type 2 simplify the rules and those make the control signal more stable against the change of input signal influenced by disturbances. On the other hand, fix form of the triangle membership of Fuzzy Type 1 fails in reducing this fluctuation because of its control signal value becomes very sensitive against, even small change of input signal.

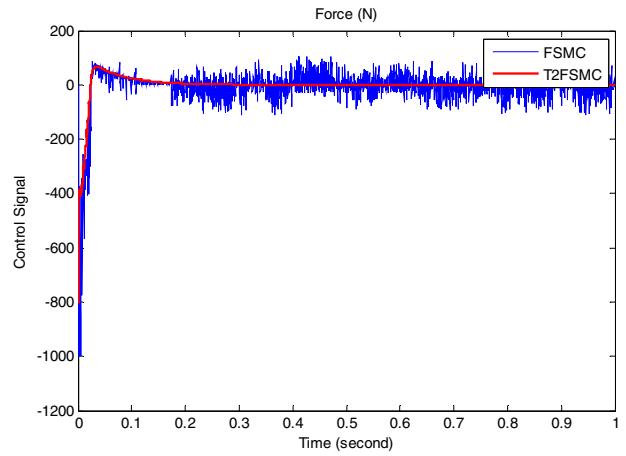


Figure 10 Control Signal of Inverted Pendulum

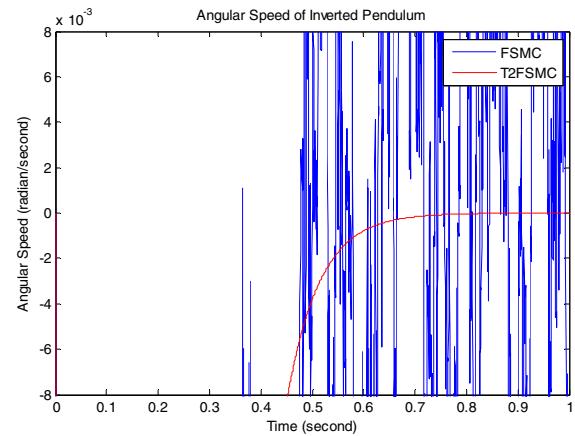


Figure 11 Angular Speed of Inverted Pendulum

Table III shows the values of ITAE of angle position of the proposed method compared with the method resulted by FSMC. Other performance indicators, such as ISE and IAE, are also presented. It can be concluded from this table that the ITAE values of the proposed method are smaller than FSMC, which guarantees that T2FSMC has a faster convergence. Fig. 12 and 13 show the comparison of response of angular speed and angle between T1FSMC and T2FSMC caused by disturbance. Although T2FSMC oscillate higher but it fastly back to the initial position without chattering.

TABLE III  
PARAMETER OF INVERTEDPENDULUM

Controller	ITAE	IAE	ISE
FSMC	0.1154	0.04718	0.01176
T2FSMC	0.002488	0.03844	0.01129

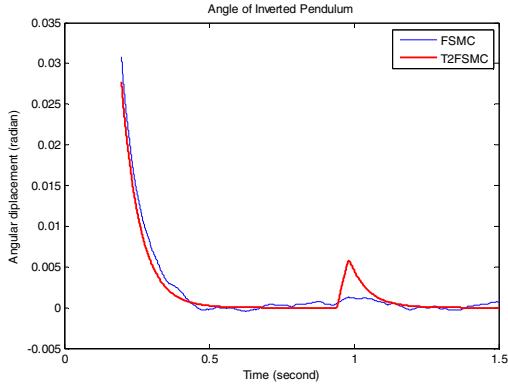


Figure 12 Angle of Inverted Pendulum with Disturbance

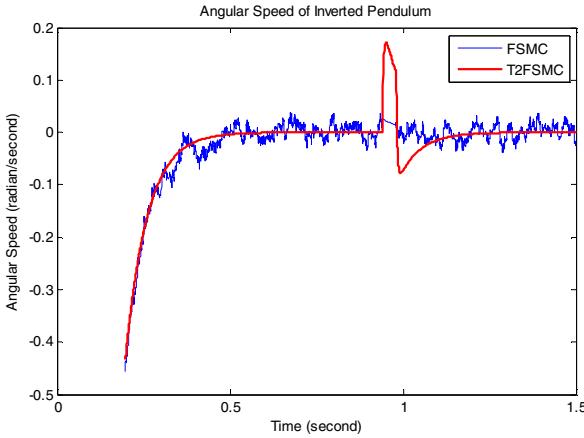


Figure 13 Angular Speed of Inverted Pendulum with Disturbance

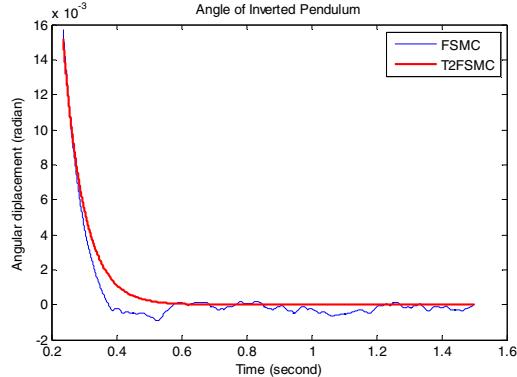


Figure 14 Angle of Inverted Pendulum with 5 % changing of Mp,Mw and L

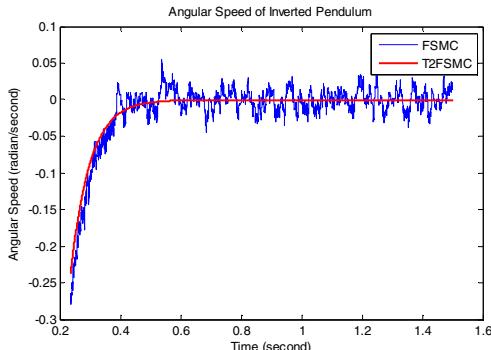


Figure 15 Angular Speed of Inverted Pendulum with 5 % changing of Mp,Mw and L

Fig. 14 and 15 show the comparison of response of angle and angular speed between T1FSMC and T2FSMC for other conditions. It is observed that the proposed method gives more robust response –in this case it converges faster in every condition.

#### IV. CONCLUSIONS

It has been shown that the proposed method (T2FSMC) give smoother chattering compared with T1FSMC. This is caused by the performance of membership uncertainty of Fuzzy Type 2 which making the control signal is not too sensitive against the surface change and its derivative. The proposed method also result more robust closed loop system against the changing parameter. The ITAE value of the controlled signal (angle), which is resulted from the proposed method, is also smaller compared to FSMC. This will guarantee a better convergence time of IT2FSMC. The only drawback of the IT2FSMC is the determination of range of the membership that still trial and error, taking quite a lot of time.

#### ACKNOWLEDGMENT

We wish a highly gratitude to Indonesian Government especially The Directorate General of High Education for graduate Scholarship (BPPS) in which we receive along our study. We also sincerely thank to Prof. Oscar Castillo from Tijuana Institute of Technology, Mexico, who permitting us to use his Fuzzy Type 2 Matlab Toolbox, to support our research. Last but not least, we thanks also to other experts who give their opinion in supporting this research.

#### REFERENCES

- [1] J. E. Slotine and S. S. Sastry, *Tracking control of non linear system using sliding surfaces with application to robot manipulator*, International Journal of Control, vol.38,no. 2,1983.
- [2] Mardlijah, Lusiana P, MH Purnomo, *Design and Performance Analysis of Speed Controller in Induction Motor with Sliding Mode Control*, Proceeding of ICRG 2010, UGM Yogyakarta
- [3] Y. K. Kim and G. J. Jeon, *Error Reduction of Sliding Mode Control Using Sigmoid-Type Nonlinear Interpolation in the Boundary Layer*, International Journal of Control, Automation, and Systems, December 2004, vol. 2,523-529
- [4] Jiwei W, Lihong X, Yunshi X., *A New design method of Fuzzy Sliding Mode Controller with faster convergence*, IEEE Int. Fuzzy Systems Conference Proceedings, 1999
- [5] Zhu, F.Q., Q.M. Winfield, A., and Melhuish, C. 2003. *Fuzzy Sliding Mode Control for Discrete Nonlinear Systems*. Transactions of China Automation Society, Vol. 22, No. 2
- [6] M.Y. Hsiao, Tzuu Hseng S, JZ Lee, CH. Chao, SH Tsai, *Design of interval type 2 fuzzy sliding mode controller*, International Journal of Information Sciences. 178 (2008)
- [7] JR Castro, Oscar Castillo, LJ Martinez, *Interval Type 2 Fuzzy Logic Toolbox*, Engineering Letters, Mexico, 2007.
- [8] J. Mendel, *Uncertain Rule-based Fuzzy Logic Systems: Introduction and new directions*, N J : Prentice Hall 2001.
- [9] Feedback Instrument Ltd., *Control in a Matlab Environment: Matlab 6.5 Version*, England. (2004).
- [10] Perruquetti, W and Barbot J.P., *Sliding Mode Control in Engineering*, Marcell Dekker, New York, 2002.