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THE NUMERICAL SOLUTION OF FREE CONVECTION FLOW OF VISCOELASTIC FLUID PAST OVER A SPHERE

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Abstract

Free convection flow is heat transfer on fluid caused by buoyancy forces because of density difference. We further use The boundary layer theory to obtain governing equations. The governing equations are further transformed into non-dimensional and then transformed into the non-similar governing equation. The non-similar governing equations are solved numerically by using explicit finite difference method. The numerical results are analyzed correlation viscoelastic and Prandtl number with velocity and temperature profile. Based on the numerical results of free convection flow of viscoelastic fluid past over a sphere, we obtained that the velocity profile decreases when we apply the increasing in the values of viscoelastic and Prandtl number. The temperature profiles increases with the increasing in the values of viscoelastic, but decreases when the values of Prandtl number increases.
Keywords: Navier-Stokes, Explicit Finite Difference Method, Viscoelastic, Prandtl number Mathematics Subject Classification: 65C20, 65M06, 76A10, 76D05, 76D10

INTRODUCTION

Heat transfer is the transfer of energy from one object to another due to temperature differences between the two objects. Generally, the heat transfer process is divided into three, i.e. conduction, convection, and radiation. Convection heat transfer is the transfer of heat from a place to another caused by the movement of fluid. Heat transfer by convection is divided into two, i.e. free and forced convection. Free convection is the transfer heat caused by buoyancy forces because of the differences in density. When a fluid exposed to heat, the fluid will expand and its density will change such that the fluid move. Part of the fluid exposed to heat, its density will becomes smaller so that the fluid move to the top and turn into a cooler fluid, then the cold fluid that its density is greater than the top will move down. Forced convection is heat transfer occurs because forced by external forces.

These last few years, topics of free convection flow specifically have been developed by several researchers such as Molla, et al., (2006), Salleh, et al., (2010b). Taher (2005) examines natural convection boundary layer flow on an isothermal sphere in presence of heat generation, assuming incompressible flow in a state then resolved by using the finite difference method Keller box. Prasad, et al., (2011) examines unsteady free convection heat and mass transfer in a walters-b viscoelastic flow past a semi-infinite vertical plate. Kasim (2014) examines the free convection flow of viscoelastic fluid which past over a sphere then numerically solved by Keller box finite difference method. However, the problems in free convection fluid flow of viscoelastic non-Newtonian fluid that past over of a sphere have not been investigate, especially convection flow in a non-Newtonian fluid with a steady state viscoelastic type, incompressible, thermal power plants, and with the completion of an explicit finite difference method with Lax-Wendroff scheme.

Non-Newtonian fluid is a fluid whose viscosity changes when there are forces acting on the fluid. This causes the viscosity of non-Newtonian fluid is not constant. Non-Newtonian fluid has several types such as: solid plastic, exponential fluid, and viscoelastic fluid. Examples of non-Newtonian fluids in their daily lives, such as paint, metal composite materials, bitumen, dough, nylon, lubricating oil, sludge, blood, liquid pharmaceuticals, pulp, etc. Viscoelastic fluid is a type of non-Newtonian fluid that viscous and elastic. Examples of these fluids are bitumen, dough, nylon, etc. Viscoelastic fluid applied in oil drilling, food and paper industry. This probem solved by using mathematical model derived from the boundary layer equations. Boundary layer is a thin layer which is near the solid surface, caused by the viscousity of fluid flow on the solid surface. Boundary layer equations are simplified from complex equations, then use for describing the characteristic of a flow. The equations derived from boundary layer equations, i.e. continuity, momentum, and energy equations.

RESEARCH METHOD

Research methods developed for accomplishing the problem free convection of viscoelastic fluid past over a sphere as below.

- Constructing mathematical modeling free convection of viscoelastic fluid past over a sphere of mass, second Newton's, and thermodynamics conservation law.
- Determining boundary condition and several related parameters such like viscoelastic number (K), Prandtl number (Pr), heat generation (γ), and Grashof number (Gr).
- Getting the mathematical model consisting of continuity, momentum, and energy equations in dimensional equations form.
- Transforming mathematical modeling dimensional into non-dimensional form, then into similar form.
- Discretization of similar mathematical modeling using Forward in Time and Centered in Space and than create a simulation program based on the discretization by using softwere *MATLAB* 2012b.
- Finding the effect viscoelastic (K) and Prandtl number (Pr) parameters on velocity profile (f') and temperature profile (θ) at lower stagnation point $(x \approx 0)$.

MATHEMATICAL MODELLING

The problems observed boundary layer are described in Figure 1.

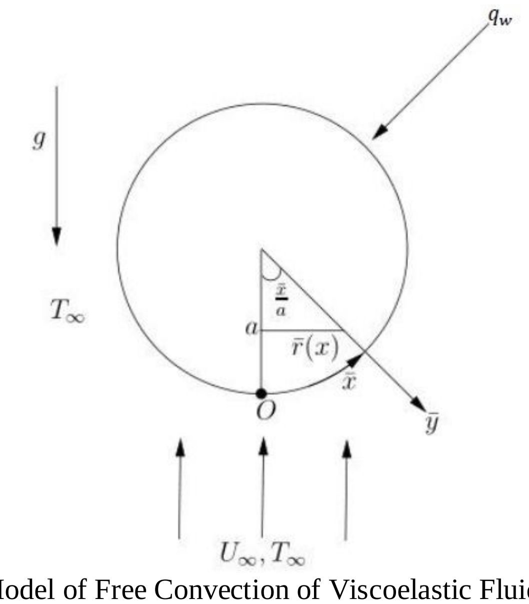

Figure 1. Physical Model of Free Convection of Viscoelastic Fluid Past Over A Sphere

Figure 1. shows shape geometry of the boundary layer problems in spherical coordinate system. The flow of fluid is assumed to move across the surface of a sphere with radius a immersed in a viscous and incompressible fluid. The ambient temperature is T_∞ and assumed that the heat flux from the surface of the sphere is q_w. Mathematical modeling free convection of viscoelastic fluid past over a sphere as below.

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\,\bar{u}) + \frac{\partial}{\partial \bar{y}}(\bar{r}\,\bar{v}) = 0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = v\left[\frac{\partial^2 \bar{u}}{\partial^2 \bar{y}}\right] + g\beta(\bar{T} - \bar{T}_\infty)\sin\left(\frac{\bar{x}}{a}\right) - \frac{k_0}{\rho}\left[\bar{u}\left(\frac{\partial^3 \bar{u}}{\partial \bar{x}\partial \bar{y}^2}\right) + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - \frac{\partial \bar{u}}{\partial \bar{y}}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right) + \frac{\partial \bar{u}}{\partial \bar{x}}\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)\right] \tag{2}$$

$$\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \alpha\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{Q_0}{\rho C_p}(\bar{T} - \bar{T}_\infty) \tag{3}$$

with boundary condition:

$$\bar{u} = \bar{v} = 0, \quad \frac{\partial \bar{T}}{\partial \bar{y}} = -\frac{q_w}{k} \quad \text{at } \bar{y} = 0, \qquad \bar{u} = 0, \frac{\partial \bar{u}}{\partial \bar{y}} = 0, T = T_\infty, \quad \text{at } \bar{y} \to \infty \tag{4}$$

with non-dimensional parameters in e.q. (5):

$$v = \frac{a}{v}Gr^{-\frac{1}{4}}\bar{v}, \theta = \frac{(\bar{T} - \bar{T}_\infty)}{\left(\frac{q_w a}{k}\right)}, x = \frac{\bar{x}}{a}, y = Gr^{\frac{1}{4}}\left(\frac{\bar{y}}{a}\right), u = \frac{a}{v}Gr^{-\frac{1}{2}}\bar{u}, r(x) = \frac{\bar{r}(\bar{x})}{a} \tag{5}$$

and than substitution to e.q. (1), (2), and (3) we obtain model of free convection of viscoelastic fluid past over a sphere in non-dimensional form, such as e.q. (6), (7), and (8).

$$\frac{\partial}{\partial \bar{x}}(ru) + \frac{\partial}{\partial \bar{y}}(rv) = 0 \tag{6}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - K\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\right] + \theta \sin x \tag{7}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + \gamma\theta \tag{8}$$

with boundary condition:

$$u = v = 0 \quad \theta' = -1 \quad \text{at} \quad y = 0, u = 0, \frac{\partial u}{\partial y} = 0, \theta = 0, \quad y \to \infty, \tag{9}$$

where $K$and Pr is viscoelastic and Prandtl number parameter, Gr and γ is Grashof number and heat generation defined as:

$$K = \frac{k_0 Gr^{5/2}}{a^2} \; and \; Pr = \frac{v}{\alpha}, \quad Gr = \frac{g\beta a q_w a^3}{kv^2} \; and \; \gamma = \frac{a^2 Q_0}{v\rho C_p Gr^{\frac{1}{2}}}$$

further, by using similari transformation function defined as:

$$\psi = xr(x)f(x,y), \qquad \theta = \theta(x,y) \tag{10}$$

where:

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y} \quad \text{dan} \quad v = -\frac{1}{r}\frac{\partial \psi}{\partial x} \tag{11}$$

and than substitution to e.q. (11) we obtain model of free convection of viscoelastic fluid past over a sphere in non-similar form, such as e.q. (12), and (13).

$$\frac{\partial^3 f}{\partial y^3}-\left(\frac{\partial f}{\partial y}\right)^2+\left(1+x\frac{\cos x}{\sin x}\right)f\frac{\partial^2 f}{\partial y^2}+\theta\frac{\sin x}{x}$$
$$-K\left[2\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}-\left(1+x\frac{\cos x}{\sin x}\right)\left(f\frac{\partial^4 f}{\partial y^4}+\left(\frac{\partial^2 f}{\partial y^2}\right)^2\right)\right]$$
$$=Kx\left[\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{\partial f}{\partial y}\frac{\partial^4 f}{\partial x\partial y^3}-\frac{\partial f}{\partial x}\frac{\partial^4 f}{\partial y^4}-\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial x\partial y^2}\right]$$
$$+x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right) \quad (12)$$

$$\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}+f\left(1+x\frac{\cos x}{\sin x}\right)\frac{\partial\theta}{\partial y}+\gamma\theta=x\left(\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial\theta}{\partial y}\right) \quad (13)$$

with boundary condition:

$$f=0,\frac{\partial f}{\partial y}=0 \quad \theta'=-1 \quad at\ y=0 \qquad \frac{\partial f}{\partial y}=0,\frac{\partial^2 f}{\partial y^2}=0 \quad \theta=0\ at\ y\to\infty \quad (14)$$

when $x\to 0$, then e.q. (12) e.q (13) become:

$$f'''-f'^2+2ff''+\theta+2K\left(ff''''-f'f'''+\left(f''\right)^2\right)=0 \quad (15)$$

$$\frac{1}{Pr}\theta''+2f\theta'+\gamma\theta=0 \quad (16)$$

with boundary condition:

$$f(0)=f'(0)=0, \qquad \theta'(0)=-1\ f'(\infty)=0,\ f''(\infty)=0, \qquad \theta(\infty)=0 \quad (17)$$

NUMERICAL SOLUTON

Mathematical model shown in equation (15) and equation (16) discretized by using explicit finite difference method with the scheme of forward time central space (FTCS) and acquired forms of discretization as follows:

$$r_2\left(\frac{1}{2}f_{i+2}-f_{i+1}+f_{i-1}-\frac{1}{2}f_{i-2}\right)+s_1\left(f_{i+1}^2+f_{i-1}^2\right)+s_2f_{i+1}f_{i-1}+s_3\left(f_if_{i+1}+f_if_{i-1}\right)$$
$$+s_4f_i^2+\theta_i+Kr_3\left[2f_if_{i+2}+2f_if_{i-2}-\frac{1}{2}f_{i+1}f_{i+2}+\frac{1}{2}f_{i+1}f_{i-2}+\frac{1}{2}f_{i-1}f_{i+2}-\frac{1}{2}f_{i-1}f_{i-2}\right]$$
$$=0 \quad (18)$$

$$\theta_i=\frac{(r_4+r_5f_i)\theta_{i+1}+(r_4-r_5f_i)\theta_{i-1}}{\gamma-2r_4} \quad (19)$$

Then by applying the Gauss Seidel iteration method in equation (18), we obtained:

$$f_i=\left[-\frac{1}{s_4}\left\{r_2\left(\frac{1}{2}f_{i+2}-f_{i+1}+f_{i-1}-\frac{1}{2}f_{i-2}\right)+s_1\left(f_{i+1}^2+f_{i-1}^2\right)+s_2f_{i+1}f_{i-1}\right.\right.$$
$$+s_3\left(f_if_{i+1}+f_if_{i-1}\right)+s_4f_i^2+\theta_i$$
$$+Kr_3\left(2f_if_{i+2}+2f_if_{i-2}-\frac{1}{2}f_{i+1}f_{i+2}+\frac{1}{2}f_{i+1}f_{i-2}+\frac{1}{2}f_{i-1}f_{i+2}\right.$$
$$\left.\left.\left.-\frac{1}{2}f_{i-1}f_{i-2}\right)\right\}\right]^{\frac{1}{2}} \quad (20)$$

where $r_1=\frac{1}{\Delta y^2}$, $r_2=\frac{1}{\Delta y^3}$, $r_3=\frac{1}{\Delta y^4}$, $r_4=\frac{1}{Pr\Delta y^2}$, $r_5=\frac{1}{\Delta y}$, $s_1=3Kr_3-\frac{1}{4}r_1$, $s_2=\frac{1}{2}r_1+2Kr_3$, $s_3=2r_1-16Kr_3$, $s_4=20Kr_3-4Kr_1$

RESULT AND DISCUSSION

The results obtained from simulation are the effect of viscoelastic parameters (K) and Prandtl number (Pr) to the velocity profile (f') and temperature profile (θ).

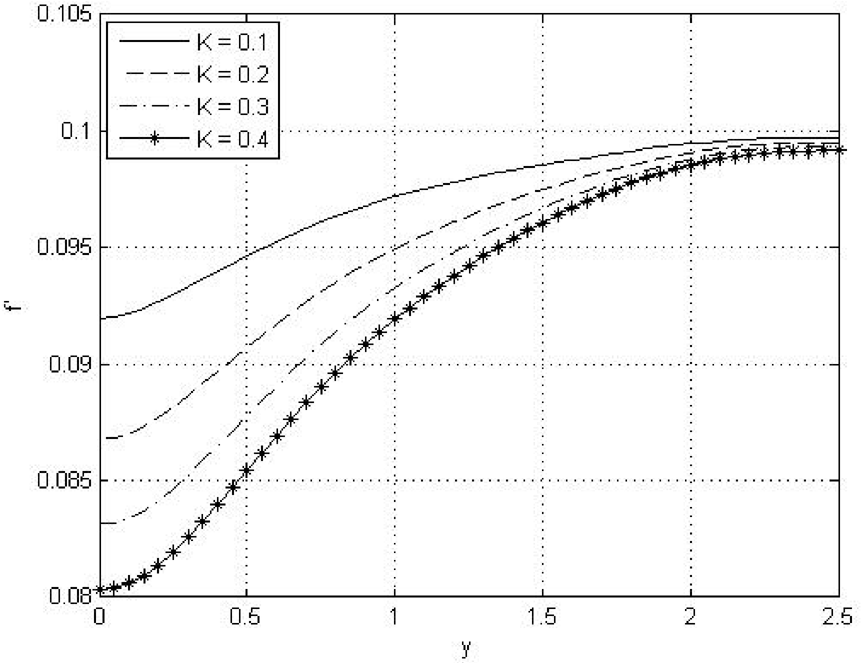

Figure 2. Velocity Profile with respect to the thickness and various K

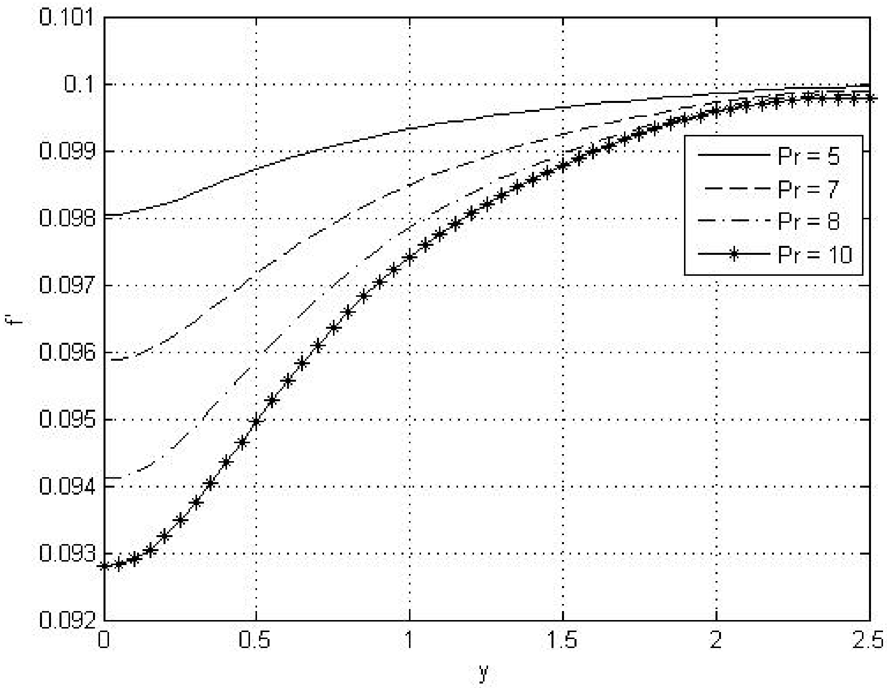

Figure 3. Velocity Profile with respect to the thickness and various Pr

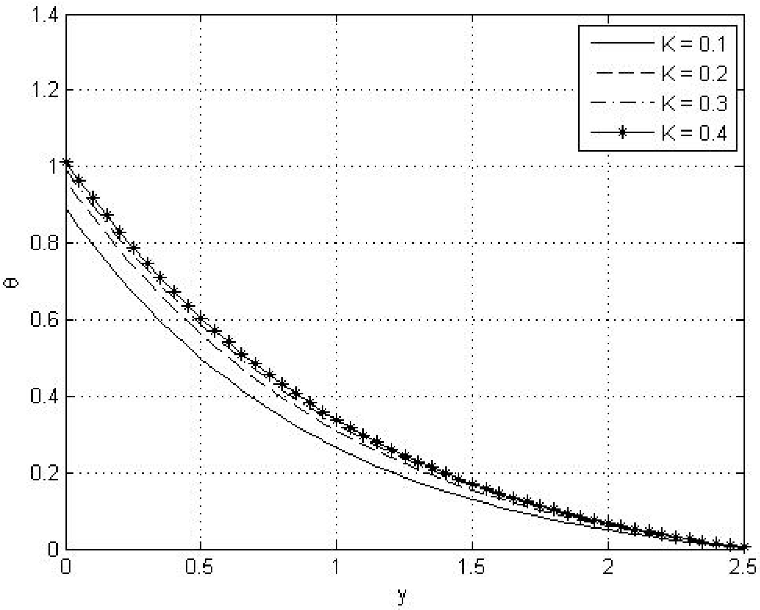

Figure 4. Temperature Profile with respect to the thickness and various K

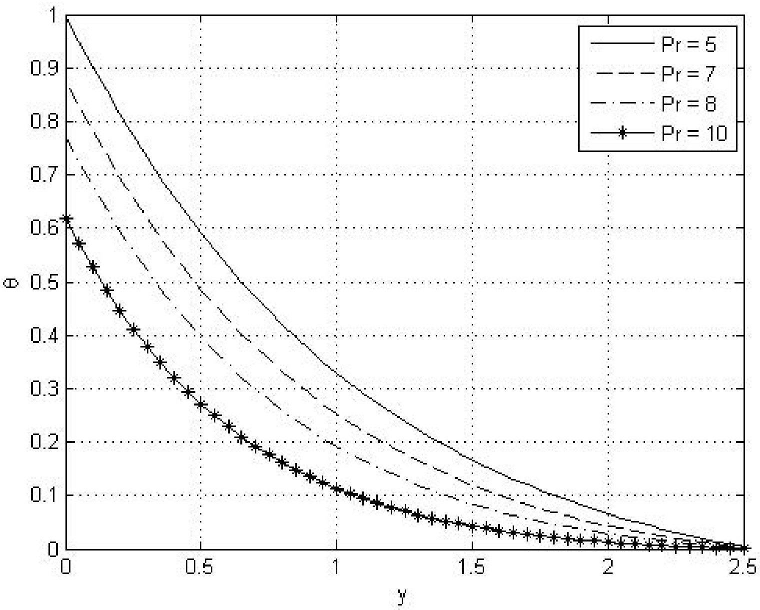

Figure 5. Temperature Profile with respect to the thickness and various Pr

Figure 2. describes the effect of viscoelastic parameters to the velocity profile. The increasing of the viscoelastic parameter, causes decreasing of the velocity of fluid flow across the surface of a sphere. This is because the viscoelastic parameters (K) shows the viscosity of a fluid, that causes a friction between the sphere with fluid, so that the fluid flow become slower. Figure 3. describes the effect of Prandtl number to the velocity profile. The increasing of Prandtl number, causes decreasing of the velocity of fluid flow across the surface of a sphere. This is because the Prandtl number is proportional to the viscosity kinematic fluid $(Pr \sim \nu)$, and the kinematic viscosity of the fluid is directly proportional to a coefficient of viscosity of the fluid

$(\nu \sim \mu_0)$, so the Prandtl number of parameters also directly proportional to the coefficient of viscosity of the fluid $(Pr \sim \mu_0)$.

Based on the results of the graph in Figure 4, the viscoelastic parameters versus proportional to the temperature profile. The increasing of viscoelastic parameter causes the increasing of the temperature profile. This is because the more viscous fluid that past over a sphere, the greater the friction between the fluid and sphere surface, then the greater fluid temperature becomes. Figure 5 describes the effect of Prandtl numbers to temperature profile. The increasing of Prandtl number causes the decreasing of the temperature profile. This is because Prandtl numbers shows temperature distribution on the free convection flow past over a sphere.

CONCLUSION

The numerical results of free convection flow of viscoelastic fluid past over a sphere indicate that the velocity profile decreases when we apply the increasing in the values of viscoelastic and Prandtl number. The temperature profiles increases with the increasing in the values of viscoelastic, but decreases when the values of Prandtl number increases.

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