

icwrdep 2015

THE 1ST YOUNG SCIENTIST INTERNATIONAL CONFERENCE
OF WATER RESOURCES DEVELOPMENT
AND ENVIRONMENTAL PROTECTION

5 - 7 June 2015

PROCEEDING

**Environmental Engineering & Water Technology
Integrated Water Systems & Governance
Water Science & Engineering**

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**WATER RESOURCES ENGINEERING DEPARTMENT
FACULTY OF ENGINEERING**

FOREWORD

The 1st Young Scientist International Conference of Water Resources Development and Environmental Protection 2015 (ICWRDEP 2015) Water Resources Engineering Department, Faculty of Engineering, University of Brawijaya was conducted on 5 - 7 June 2015. The Conference was organized by Faculty of Engineering and collaborated with International University of Malaya (UM), Universiti Sains Malaysia (USM) and Universiti Tun Hussein Onn Malaysia (UTHM).

The participants of the Conference are about 60 participants come from more than 20 higher institutions, such as; Sepuluh Nopember Institute Of Technology, Surabaya (ITS), Bandung Institute of Technology (ITB), Bogor Agricultural University (IPB), The University of Lampung, Sriwijaya University, University of Muhammadiyah Malang (UMM), University of Brawijaya (UB), Padjajaran University, State University of Malang (UM), National Institute of Technology (ITENAS), Tidar university, State Polytechnic of Malang (Politeknik Negeri Malang), Mulawarman University, State Polytechnic of Padang (Politeknik Negeri Padang), Malang National Technology Institute (Institut Teknologi Nasional Malang), BBWS Mesuji Sekampung, Bengkulu University, Diponegoro University (UNDIP), Nusa Cendana University, Khairun University, Bantara University, University of Jember, State Polytechnic of Samarinda (Politeknik Negeri Samarinda), UM (University of Malaya), Universiti Sains Malaysia (USM) and Universiti Tun Hussein Onn Malaysia (UTHM), and others, which reflect the importance water resources engineering development and environmental protection.

The topics of conference are Environmental Engineering & Water Technology, Integrated Water System & Governance and Water Science & Engineering. The conference provide platform for researchers, engineers and academician to meet and share ideas, achievement as well as experiences through the presentation of papers and discussion. These events are important to promote and encourage the application of new concept of water resources development and techniques to practitioners as well as enhancing the knowledge of environmental protection with the current requirements of analysis, design and construction of any engineering concept.

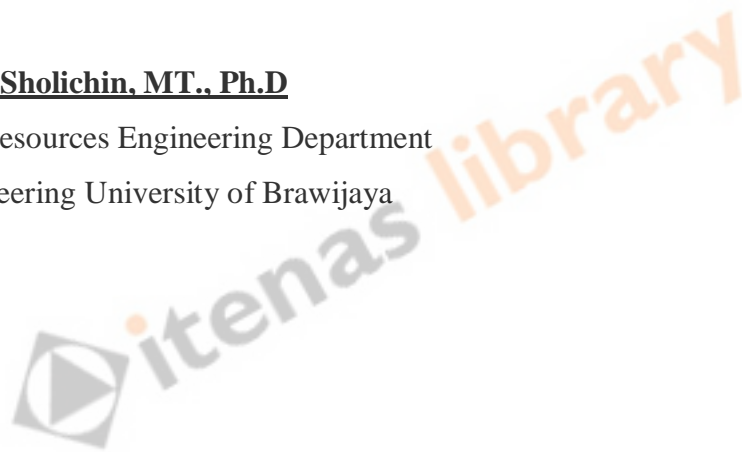
As Head of Water Resources Engineering Department, we would like to express our deepest gratitude to the Rector University of Brawijaya, Keynote Speakers (Prof Satoru Oishi & Prof Tsuyoshi Imai from Japan, Assoc. Prof Faridah Othman and Prof Amir Hamzah from Malaysia), International Advisory Board members, organizing committee and also to all participants.

We would like to express our deepest gratitude to the Faculty of Engineering conducted such conference. This is the first International conference for the Department and we expect that this is will become 2nd annual activity for our Department.

Malang, 5 June 2015

Ir. Mohammad Sholichin, MT., Ph.D

Head of Water Resources Engineering Department
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Table of Content

	Page
Foreword.....	i
Editorial Boards	iii
Editorial Reviewers	iii
 THEME 1 Environmental Engineering & Water Technology	
Circulation Effect Of Coffee Wastewater Flow In Water Hyacinth	
Phytoremediation	A-1
Elida Novita, Sri Wahyuningsih, Siswoyo Soekarno, Betty Siska Rukmawati	
 Potential Greywater Quantification For Reuse In Newton Residence Apartment	
Bandung, Indonesia.....	A-8
Dyah Asri Handayani Taroepratjeka, Yulianti Pratama, Devi Ayu Putrianti	
 Analyzing Water Quality Changes Due To Agriculture Activities In Seputih	
Irrigation Area, Lampung Province, Indonesia.....	A-15
Eka Desmawati, Rusdi Effendi, Yudha Mediawan, Gatot E. Susilo	
 Evaluation of Environmentally Friendly Flushing in Wlingi and Lodoyo	
Reservoirs	A-23
Fahmi Hidayat	
 Dynamic of Dissolved Oxygen At Inlet Zone Of Fish Cage Area In Cirata Reservoir,	
West Java, Indonesia.....	A-30
Fanny Novia, Priana Sudjono, Arief Sudrajat	
 Intensive Agriculture of Peat Land Areas To Reduce Carbon Emission And Fire	
Prevention (A Case Study In Tanjung Jabung Timur Tidal Lowland Reclamation	
Jambi)	A-38
Momon Sodik Imanudin1, and R.H Susanto	

**Mikro-Nano Activated Charcoal from Ricestraw as Adsorben Heavy Metals Leachate,
Case Studies on “TPA JATIBARANG”, Semarang Jawa Tengah A-49**

Rizki Januarita, Anis Ulfa W.A, Azka Azizah, Hilma Muthi'ah

**Determination of Water Quality Status at Karang Mumus River Samarinda,
Indonesia A-59**

Sri lestari, Diana Arfiati, Aniek Masreवानiah, Moch. Sholichin

**Efficiency Analysis of Cod And Bod Decline Wastewater Coffee On Phytoremediation
Process Using Water Hyacinth (Eichornia Crassipes (Mart.) Solms) A-62**

Setyorini, Sri Wahyuningsih, Elida Novita

Green Roof: Vegetation Response towards Lead and Potassium..... A-69

Khairul Rahmah Ayub, Aminuddin AB Ghani, Nor Azazi Zakaria

**Water Content – Density Criteria of Bentonite – Fly Ash Mixtures for Compacted Soil
Liners A-77**

Andre Primantyo Hendrawan, Dian Chandrasasi1, Runi Asmaranto, Anggara Wiyono Wit Saputra, Linda
Irnawati Gunawan, Zaenal Abidin

THEME 2 Integrated Water Systems & Governance

**Experience in Rainwater Harvesting Application For Household Scale In Bandar
Lampung, Indonesia B-1**

Gatot Eko Susilo

**Estimation of the Flood Using Data Modis to Support Integrated Water Resources
Management B-9**

Gusta Gunawan, Alex Surapati, Besperi

**Alternative Selection for Water Resource Potential in Brantas Watershed
For The Development of Hydroelectric Power Plant..... B-16**

Deviany Kartika, Miftahul Arifin

Analysis Availability on the Clean Water Infrastructure at PDAM Ternate B-23

Nani Nagu

Rainfall Estimation Using Weather Radar and the Flood Simulation at Ciliwung River Indonesia Analysis	B-30
Ratih Indri Hapsari, Agus Suhardono, Reni Sulistyowati	
Integrated Coastal Zone Management with Watershed Management Based On Co-Management: A Case Study Porong River Along Sidoardjo-Pasuruan Coastal Area	B-37
Rudianto	
The Evaluation of Song Bajul Springs Potency For Resident's Clean Water Supply In Desa Pucanglaban Kecamatan Pucanglaban Kabupaten Tulungagung In 2015-2030.....	B-46
Sam Yudi Susilo, Hendra Agus	
Flow Analysis On Pipe Distribution Network Using Differential Evolution Algorithm (DE).....	B-54
Sulianto	
Hydroinformatics In Volumetric And Real Time Irrigation Discharge Monitoring.....	B-63
Susi Hidayah, Aditya Prihantoko, and Irfan Sudono	
Multiple Stacked Rule Curves For Reservoir Operation Of Medium Reservoir	B-71
Widandi Soetopo, Lily Montarcih Limantara, Suhardjono, Ussy Andawayanti, Rahmah Dara Lufira	
Water Balance Analysis Due To the Human Live Requirements.....	B-76
Agus Suharyanto, Very Dermawan, Mustika Anggraeni, Pudyono , Kurniawan Sigit Wicaksono, Diah Susilowati	
Optimization System Network Providing Water Study Blitar District Of Kademangan East Java Indonesia.....	B-84
Rahmah Dara Lufira, Suwanto Marsudi, Jadfani Sidqi F., Evi Nur Cahya	
Safety Inspection of Prijetan Dam.....	B-89
Runi Asmaranto	

Analysis of Conditions Changes In Sumi Dam Hydrology Parameters

Design B-100

Anggara WW. Saputra

THEME 3 Water Science & Engineering

Investigation of Marine Debris In Kuta Beach, Bali..... C-1

Adli Attamimi, Noir P. Purba, Santi R. Anggraini, Syawaludin A. Harahap

Design of Marine Propulsion System Based On Structural Vibration..... C-8

Asep Andi, Radite Praeko Agus Setiawan

Transmission and Wave Reflection on Double Submerged Breakwater C-16

Bambang Surendro

Calibration of Measurement on Modelling Stepped Spillway C-24

Denik Sri Krisnayanti, Soehardjono, Moch. Sholichin, Very Dermawan, Nina B. Rustiati

Estimates of Time of Concentration in Rainfall, Runoff and Infiltration

Application C-33

Dian Noorvy, Lily Montarcih, Donny Harisuseno

Comparing the Calculation Method of the Manning Roughness Coefficient in Open

Channels C-42

Hari Wibowo

Grouping Watersheds Through Hierarchical Clustering Approach C-53

Judi K. Nasjono, Mohammad Bisri, Agus Suharyanto, Dian Sisanggih

**Study on the Effectivity of Decreasing Permeability and Increasing Shear Strength of
Sandy Beach Soil And River Soil By Using Exopolysaccharide Biopolymer** C-62

Emma Yuliani, Maytri Handayani, Ariska Desy Haryani

Heat Effect on Fluid Free Convection Flow Past A Porosity Sphere C-70

Mohamad Tafrikan, Basuki Widodo, Chairul Imron

Incompressible and Steady Mixed Convection Flow Past Over a Sphere C-78

Mohammad Ghani, Basuki Widodo, Chairul Imron

Viscoelastic Fluid Past a Flat Plate with the Effect of Magneto hydrodynamic C-85

Putri Pradika Wanti, Basuki Widodo, Chairul Imron

Flow Measurement Under Sluice Gate Model C-94

Rustiati, N.B., Suhardjono, Rispiningtati, Dermawan, V., Krisnayanti, D.S

Kinetic Modeling of Domestic Wastewater (Greywater Type) Using Uasb

Reactor C-102

S. Syafrudin, P. Purwanto, S. Sudarno

An Imaging Technique for Identifying Flow Structure and Magnitude In

A Channel C-113

Tommy E. Sutarto, Habir, S.S.N. Banjarsanti

**The Numerical Solution Of Free Convection Flow of Visco-Elastic Fluid With Heat
Generation Past Over A Sphere C-122**

Wayan Rumite, Basuki Widodo, Chairul Imron

**Assessment of Sedimentation Patterns and the Threat of Flooding due to Reclamation in
The Lamong Bay, Indonesia C-128**

Mohammad Sholichin, Tri Budi Prayogo, Sebrian Mirdeklis Beselly Putra, Rini Wahyu Sayekti

**Design Improvements To The Physical Model Test Spillway Of Mujur Dam In Lombok
Tengah Region C-145**

Dian Chandrasasi, Dwi Priyantoro, Anggara WW. Saputra

Hydropower Plant using Pump storage at Cisokan Dam C-151

Endang Purwati

Model Test of Physical Spillway In Lesti Dam, Malang District East Java C-155

Heri Suprijanto, Janu Ismoyo, Sumiadi, Yuli Astuti

**A Network Rain Station in Reviewed of the Topography on Watershed Widas District
Nganjuk – East Java of Indonesia C-163**

Eri Prawati, Suhardjono, Lily Montarjih, Rispiningtati

Application of Design Charts for Determination of Landfill Liner's Thickness ... C-170

Andre Primantyo Hendrawan, Anggara Wiyono Wit Saputra, Runi Asmaranto, Dian Chandrasasi, Hestina
Eviyanti, Zaenal Abidin





**Environmental Engineering &
Water Technology**



Incompressible And Steady Mixed Convection Flow Past Over A Sphere

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ABSTRACT

Mixed convection flow is the combination between free convection flow and forced convection flow. In this research, the pressure and buoyancy forces are significant. The governing equations are taken from Navier-Stokes equation that includes continuity, momentum, and energy equations. These equations are obtained from Boussinesq and boundary layer approximations. These non-dimensional equations are then transformed into non-dimensional equations to make easy in numerical processes. Further, these non-dimensional equations are transformed into non-similar equations and solved numerically using the finite difference method. The numerical results are analyzed the effect of Prandtl parameter (P_r) and visco-elastic parameter (K) to velocity profile (f') and temperature profile (θ). The temperature profile decreases and the velocity profile decreases when the prandtl parameter increases. Meanwhile, the temperature profile increases and the velocity profile decreases when the visco-elastic parameter increases.

KEYWORDS

Navier-Stokes, Mixed Convection Flow, Visco-elastic Fluid, Boundary Layer Theory

INTRODUCTION

The boundary layer problems of mixed convection flow past over a sphere are fundamental theory and have been applied widely in engineering applications. Many researchers have investigated these problems in different geometries such as flat plate, cone, and cylinder with type of fluids Newtonian or non-Newtonian. Boundary layer on fluid is a layer near surface of medium so the effect of viscosity and velocity profile to be significant because of shear stress at the wall (Sleigh and Andrew, 2001). In this research, the mixed convection flow that is the combination between free convection flow and forced convection flow is analyzed (Kreith and Frank, 1994). The researches of mixed convection past over a sphere have been studied by several researchers such as Amin et al (2002) studied mixed convection flow past over a surface of sphere in steady state and incompressible with the constant temperature. Further, the numerical solutions were solved by the Box-Keller method. Nazar et al (2010) studied mixed convection flow past over a sphere with Newtonian heating. Heat transfer of Newtonian heating was proportional to local surface temperature. Salleh and Ibrahim (2002) studied mixed convection flow past over a sphere at lower stagnation point with Newtonian heating. Temperature profile and velocity profile were analyzed based on mixed convection parameter and Prandtl number. Kasim (2014) studied mixed convection flow of viscoelastic fluid past over a sphere in steady-state and incompressible that was solved numerically by the Box-Keller method. Based on the previous researches, this research is studied mixed convection flow of visco-elastic fluid past over a surface of sphere with the effect of magnetohydrodynamics in steady state and incompressible. These non-similar equations are solved numerically using the finite difference method with iterative method to solve non-linear ordinary differential equations. In this research, it is only investigated laminar flow of visco-elastic fluid past over a sphere surface. This means that the velocity of fluid is

small because of the visco-elastic effect that is shown by the Reynolds number $R_s < 500$ (Widodo, 2012).

MATHEMATICAL MODELLING

Consider steady-state two-dimensional mixed convection flow of visco-elastic incompressible fluid past over a sphere with the effect of magnetohydrodynamic (MHD) where a is radius of sphere. The physical model of this research is illustrated as follows.

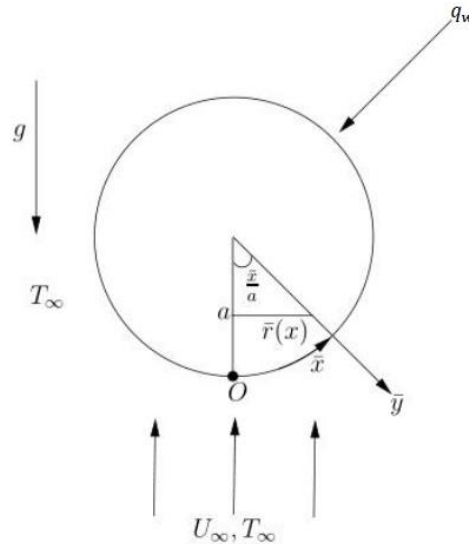


Figure 1. Physical Model of Free Convection of Visco-elastic Fluid Past Over a Sphere

Figure 1 gives illustration of the physical model and coordinate system on mixed convection flow of visco-elastic fluid past over a sphere surface. It assumed that q_w is heat flux of sphere surface and T_∞ is temperature of visco-elastic fluid. Based on the Boussinesq and boundary layer approximations, then obtained the following basic equations of continuity, momentum, and energy equations that have been studied by Widodo (2013) and Kasim (2014).

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\bar{u}) + \frac{\partial}{\partial \bar{y}}(\bar{r}\bar{v}) = 0 \quad (1)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \bar{u}_s\frac{\partial \bar{u}_s}{\partial \bar{x}} + \bar{v}\left[\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}\right] - \frac{k_0}{\rho}\left[\bar{u}\left(\frac{\partial^3 \bar{u}}{\partial \bar{x}^3 \partial \bar{y}^2}\right) + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} + \frac{\partial \bar{u}}{\partial \bar{x}}\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)\right] + \frac{k_0}{\rho}\left[\frac{\partial \bar{u}}{\partial \bar{y}}\left(\frac{\partial^2 \bar{u}}{\partial \bar{y} \partial \bar{x}}\right)\right] - g\beta(\bar{T} - \bar{T}_\infty)\sin\left(\frac{\bar{x}}{\bar{a}}\right) - \frac{1}{\rho}\sigma(\bar{u} - \bar{u}_s)B_0^2 \quad (2)$$

$$\left(\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}\right) = \alpha\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \quad (3)$$

with the following boundary conditions.

$$\bar{u} = \bar{v} = 0, \frac{\partial \bar{T}}{\partial \bar{y}} = -\frac{q_w}{k} \text{ at } \bar{y} = 0 \text{ and } \bar{u} = \bar{u}_s(x), \frac{\partial \bar{u}}{\partial \bar{y}} = 0, T = T_\infty \text{ at } \bar{y} \rightarrow \infty \quad (4)$$

where $u_s(x)$ is velocity of local free flow at the outside of boundary layer that is defined by $u_s(x) = \frac{3}{2}U_\infty \sin\left(\frac{\bar{x}}{\bar{a}}\right)$. Further, the non-dimensional variables are introduced as follows.

$$x = \frac{\bar{x}}{a}, y = R_s^2\left(\frac{\bar{y}}{a}\right), r(x) = \frac{\bar{r}(\bar{x})}{a}, u = \frac{\bar{u}}{U_\infty}, v = R_s^2\left(\frac{\bar{v}}{U_\infty}\right), \theta = \frac{R_s^2(T - T_\infty)k}{q_w a}, u_s(x) = \frac{\bar{u}_s(\bar{x})}{U_\infty} \quad (5)$$

By substituting Equation (5) into Equations (1) to (3), then obtained the following non-dimensional equations.

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \quad (6)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_s \frac{\partial u_s}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \lambda \theta \sin(x) - K \left[v \frac{\partial^3 u}{\partial y^3} + u \frac{\partial^3 u}{\partial x \partial y^2} \right] + K \left[\frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} \right] - M(u - u_s) \quad (7)$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \quad (8)$$

where K and λ are non-dimensional parameters of visco-elastic and mixed convection respectively that are defined as $K = \frac{k_0}{\rho} \left(\frac{U_\infty}{av} \right)$ and $\lambda = \frac{Gr}{Re^2}$ respectively with the following boundary conditions.

$$u = v = 0, \theta' = -1 \text{ at } y = 0 \text{ and } u_s = \frac{3}{2} \sin(x), \frac{\partial u}{\partial y} = 0, \theta = 0 \text{ at } y \rightarrow \infty \quad (9)$$

Further, according to Equation (9), then Equations (6) to (8) are solved using the following stream function

$$\psi = xr(x)f(x,y), \theta = \theta(x,y) \quad (10)$$

where ψ is defined as

$$u = \frac{1}{r} \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{1}{r} \frac{\partial \psi}{\partial x} \quad (11)$$

Based on Equation (11), then Equations (6) to (8) are written as the following non-similar equations.

$$\left(\frac{\partial^3 f}{\partial y^3} \right) + \left(1 + x \frac{\cos(x)}{\sin(x)} \right) f \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y} \right)^2 + \frac{9}{4} + \lambda \theta \frac{\sin(x)}{x} - 2K \left[\frac{\partial f}{\partial y} \frac{\partial^3 f}{\partial y^3} \right] + K \left[\left(1 + x \frac{\cos(x)}{\sin(x)} \right) \left(f \frac{\partial^4 f}{\partial y^4} + \left(\frac{\partial^2 f}{\partial y^2} \right)^2 \right) \right] = x \left(\frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y^2} \frac{\partial f}{\partial x} \right) - M \frac{\partial f}{\partial y} + \frac{3}{2} M \frac{\sin x}{x} + Kx \left[\frac{\partial^3 f}{\partial y^3} \frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^4 f}{\partial y^4} \frac{\partial f}{\partial x} - x \frac{\partial^2 f}{\partial y^2} \frac{\partial^3 f}{\partial x \partial y^2} + \frac{\partial f}{\partial y} \frac{\partial^4 f}{\partial x \partial y^3} \right] \quad (12)$$

$$x \left(\frac{\partial f}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x} \frac{\partial \theta}{\partial y} \right) = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + \left(1 + x \frac{\cos(x)}{\sin(x)} \right) f \frac{\partial \theta}{\partial y} \quad (13)$$

with the following boundary conditions.

$$f = 0, \frac{\partial f}{\partial y} = 0, \theta' = -1 \text{ at } y = 0 \text{ and } \frac{\partial f}{\partial y} \rightarrow \frac{3 \sin x}{2x}, \frac{\partial^2 f}{\partial y^2} = 0, \theta \rightarrow 0 \text{ at } y \rightarrow \infty \quad (14)$$

At the lower stagnation point ($x \approx 0$), then Equations (12) to (13) are written as

$$f'''' + 2ff''' - f'^2 + \frac{9}{4} + \lambda \theta + 2K(ff'''' - f'f'''' + f''^2) - M \left(f' - \frac{3}{2} \right) = 0 \quad (15)$$

$$\frac{1}{Pr} \theta'' + 2f\theta' = 0 \quad (16)$$

with the following boundary conditions.

$$f(0) = f'(0) = 0, \theta'(0) = -1 \text{ at } y = 0 \text{ and } f' \rightarrow \frac{3}{2}, f'' = 0, \theta \rightarrow 0 \text{ at } y \rightarrow \infty \quad (17)$$

NUMERICAL SOLUTION

Equations (15) and (16) are discretized using the finite difference method, then obtained

$$f_i = \text{sqrt} \left[(-1) * (s_1(f_{i+2} - 2f_{i+1} + 2f_{i-1} - f_{i-2}) + t_1 f_i f_{i+1} + t_3 f_i f_{i-1} + t_4 f_{i+1}^2 + t_6 f_{i-1}^2 + t_5 f_{i+1} f_{i-1} + \frac{9}{4} + \lambda \theta_i + t_7 f_i f_{i+2} + t_8 f_i f_{i-2} + t_9 f_{i+1} f_{i+2} + t_{10} f_{i+1} f_{i-2} + t_{11} f_{i-1} f_{i+2} + t_{12} f_{i-1} f_{i-2}) / t_2 \right] \quad (18)$$

where $s_1 = \frac{1}{2\Delta y^3}, s_2 = \frac{2}{\Delta y^2}, s_3 = \frac{1}{4\Delta y^2}, s_4 = \frac{1}{\Delta y^4}, s_5 = \frac{1}{4\Delta y^4}, s_6 = \frac{1}{\Delta y^4}, t_1 = s_2 - 8Ks_4 - 8Ks_6,$

$$t_2 = -2s_2 + 12Ks_4 + 8Ks_6, t_3 = s_2 - 8Ks_4 - 8Ks_6,$$

$$t_4 = -s_3 + 4Ks_5 + 2Ks_6, t_5 = 2s_3 - 8Ks_5 + 4Ks_6, t_6 = -s_3 + 4Ks_5 + 2Ks_6, t_7 = 2Ks_4, t_8 = 2Ks_4, t_9 = -2Ks_5, t_{10} = 2Ks_5, t_{11} = 2Ks_5,$$

$$t_{12} = -2Ks_5$$

$$\theta_i = \frac{[(r_1 + r_2 f_i)\theta_{i+1} + (r_1 - r_2 f_i)\theta_{i-1}]}{2r_1} \quad (19)$$

where $r_1 = \frac{1}{Pr \Delta y^2}$ and $r_2 = \frac{1}{\Delta y}$.

RESULTS AND DISCUSSION

The numerical results of this research are the effect of Prandtl number and visco-elastic parameter to temperature profile (θ) and velocity profile (f').

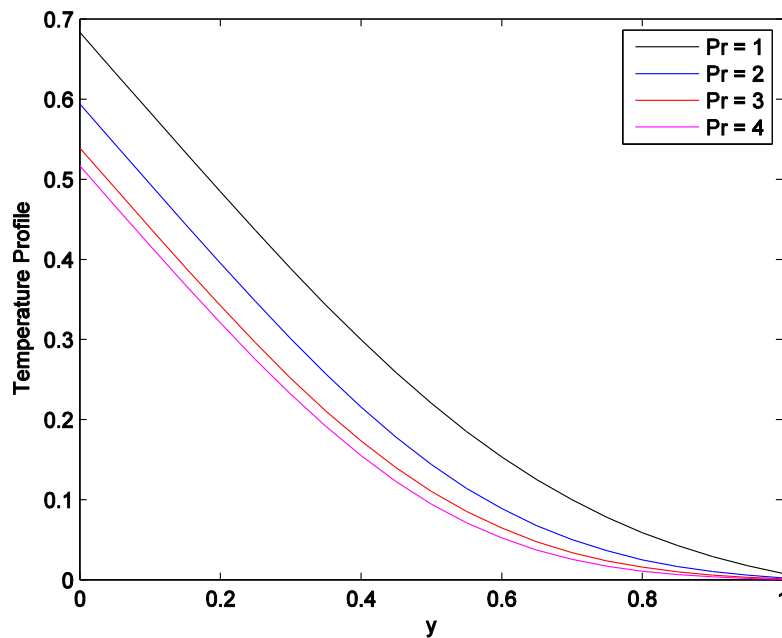


Figure 2. Prandtl number variation (Pr) for Temperature Profile (θ) with the boundary layer thickness of y

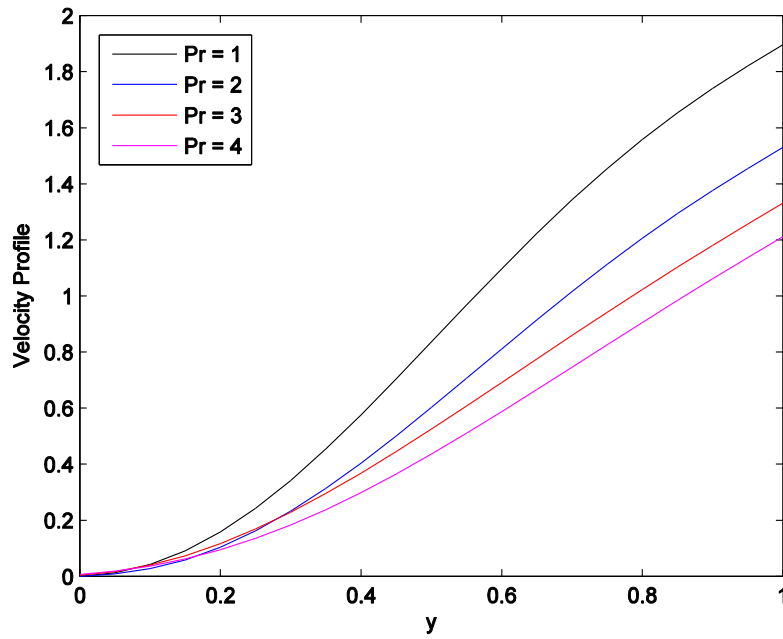


Figure 3. Prandtl number variation (Pr) for Velocity Profile (f') with the boundary layer thickness of y

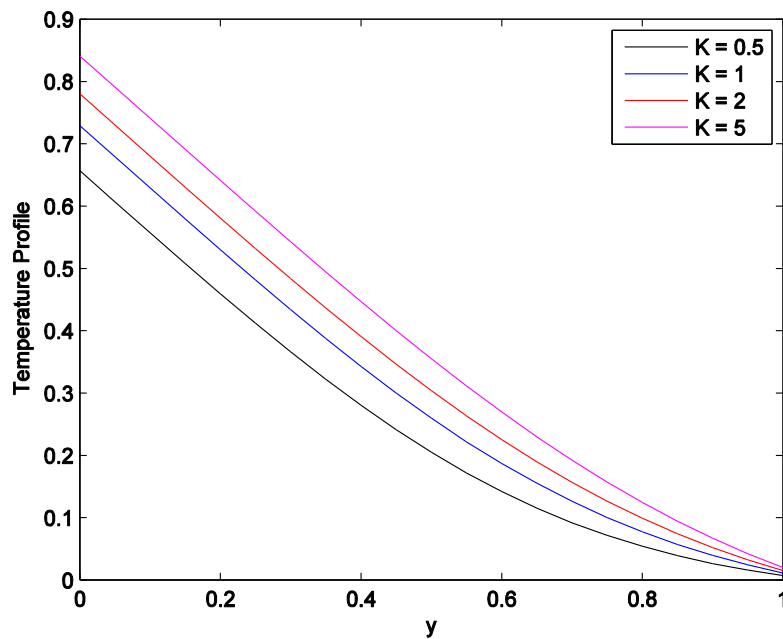


Figure 4. Visco-elastic parameter variation (K) for Temperature Profile (θ) with the boundary layer thickness of y

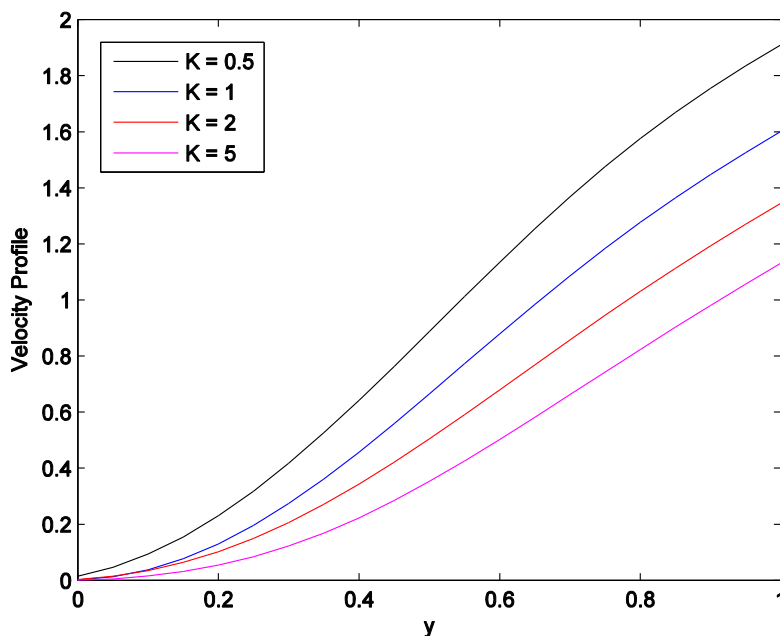


Figure 5. Visco-elastic parameter variation (K) for Velocity Profile (θ) with the boundary layer thickness of y

Figure 2 shows the effect of Prandtl number to temperature profile (θ). In this case, Prandtl number is related to heat distribution, so that when Prandtl number increases then heat distribution increases. It causes temperature profile decreases because of the increased heat distribution. The result in Figure 3 is caused by the increased heat distribution, so that the decreased temperature profile (f') causes density of visco-elastic fluid more increased. In this case, fluid flow is downward because of gravitation, so that the velocity profile is more decreased. Figure 4 shows the effect of visco-elastic parameter to temperature profile (θ). This indicates that temperature profile is more increased when the visco-elastic parameter is more increased. This is caused, the more increased visco-elastic parameter causes the friction between fluid and the sphere surface more increased, so that the temperature on sphere surface is more increased because of the more increased friction. Meanwhile, Figure 5 shows the effect of visco-elastic parameter to velocity profile (f'). The velocity profile is more decreased when visco-elastic parameter is more increased. This is caused by the more increased friction between viscoelastic fluid and sphere surface, so that the velocity profile is more decreased.

CONCLUSIONS

In this research, the problem of mixed convection flow on visco-elastic fluid past over a sphere surface with the effect of magnetohydrodynamic (MHD) has been studied. The non-similar equations of momentum and energy are solved numerically using the finite difference method with the iterative method. The effect of Prandtl number and visco-elastic parameter to the characteristic of temperature profile (θ) and velocity profile (f') have been obtained and discussed. Then, the conclusions of this research can be written as follows.

1. The temperature profile decreases and the velocity profile decreases when Prandtl number variation is more increased.
2. The temperature profile increases and the velocity profile decreases when visco-elastic parameter variation is more increased.

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