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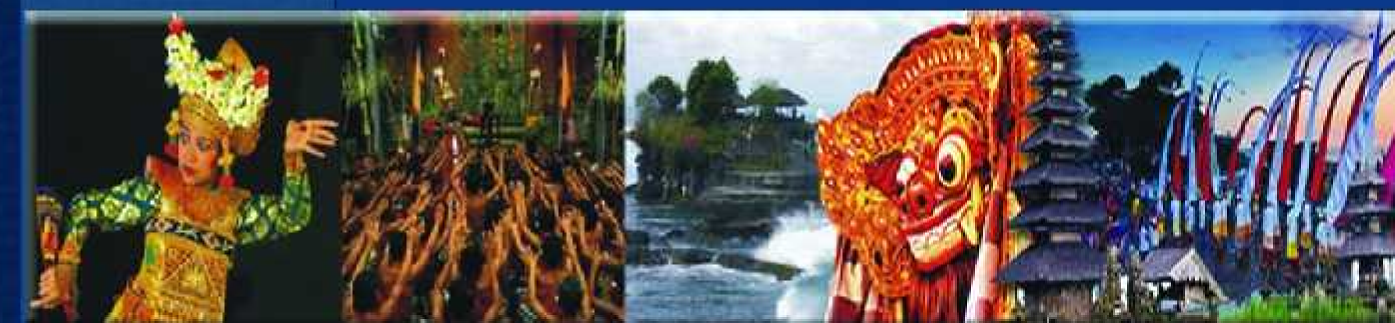

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Message

from the

Rector

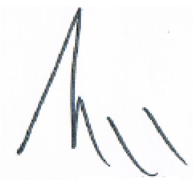
Institut Teknologi Sepuluh Nopember

I would like convey my sincere congratulation to all involved parties for the successful organization of the IMT-GT International Conference on Mathematics, Statistics and its Applications. The ICMSA is annually conference organized by The Indonesia-Malaysia-Thailand Growth Triangle (IMT-GT), and this year ITS hosts the conference, organized by the Department of Mathematics and Department of Statistics. I would like also to express my deep appreciation to Department of Mathematics, Udayana University Bali for the collaboration. This ICMSA 2012 is held as part of our 52^{nd} Institute Anniversary.

It is great pleasure for me to welcome and thank all keynote speakers for the worthy time to share your experience and expertise to all conference participants. I do believe that your participation to this conference is a highlight and give a significant insight to all of us. I expect that your patronage and support towards the advancement of knowledge through this event, will contribute to the future development of Mathematics and Statistics.

As we know that the role of Mathematics and Statistics is vital in many aspects of live. This conference is a means to share and discuss a new knowledge and inventions among researchers, practitioners and students that may lead to a more real contribution of Mathematics and Statistics in solving problems arises in social, business, economic, environment, and many others.

Last but not least, I wish all participants have a very pleasant and valuable moment during the conference. Moreover, I hope that new collaborations among participants could be established. To our foreign guests, I wish you a memorable stay in Bali. We welcome you anytime to visit our university, Institut Teknologi Sepuluh Nopember (ITS) in Surabaya.



Prof. Dr. Ir. Triyogi Yuwono, DEA.
Rector of the Institut Teknologi Sepuluh Nopember (ITS) Indonesia

Message

from the

Dean Faculty of Mathematics and Natural Sciences

Institut Teknologi Sepuluh Nopember

On behalf of the Faculty of Mathematics and Natural Sciences, Institut Teknologi Sepuluh Nopember, it is a great honor and sincere to welcome all participants to the 8th IMT-GT International Conference on Mathematics, Statistics, and it's Applications (ICMSA 2012).

This year, Department of Mathematics and Department of Statistics, Institut Teknologi Sepuluh Nopember collaborate with Department of Mathematics, Udayana University, have honor to organize this meaningful international conference. I believe that the purpose of this conference is not only sharing knowledge among mathematician, statisticians, and scholars in related fields but also to hearten new generation of expertise in mathematics and statistics to realize the science and technology advancement.

It is undeniable that science and technology are the products of mathematics and statistics applications. Many disciplines like engineering, computer science, information technology, operational research, logistics management, risk management and many others are all the products of mathematics and statistics. Thus, it is essential that we must hold this annual conference as a stage for all scholars in finding new ideas and applications on Mathematics and Statistics.

Greatly thank to all supportive session including organizing committee, keynote speakers, invited speakers, paper reviewers, participants and sponsors. This event will not achieve without you all. Finally, I hope that the outcome of ICMSA 2012 will be pleasing and most useful to everybody.

Sincerely yours,

Prof. Dr. R.Y. Perry Burhan
Dean

Message
from the
Chairman
Organizing Committee

On behalf of the organizing committee, it is my great pleasure to welcome all participants of the 8th IMT-GT 2012 International Conference on Mathematics, Statistics, and it's Applications (ICMSA 2012). This conference had been held for seven times in Indonesia, Malaysia and Thailand. It is the fourth time that Indonesia hosts the conference and Department of Mathematics and Department of Statistics, Institut Teknologi Sepuluh Nopember collaborate with Department of Mathematics, Udayana University, are honored to organize this important event.

The theme of our conference is "Mathematical and Statistical Thinking for Technology Development" highlighting the importance of mathematical and statistical science as the major tools for solving problems and making right decisions. They play vital roles to the development of science and technology in the IMT-GT region and beyond. The regular meeting among researchers in the fields like this conference will promote the progress and advancement of the fields. This conference will surely serve as a venue for researchers in the fields to present their works, exchange ideas and seek collaboration. Participants from the IMT-GT region and many countries around the world will attend the conference. More than 10 distinguished speakers from many countries are invited to give talks in the conference. So, I hope all participants will enjoy attending to the talks and paper presentations as well as have very fruitful discussions.

I would like to take this opportunity to thank all the keynote and invited speakers for coming and sharing their knowledge with us. I am also very grateful to all international and local scientific committee and many others who have contributed to the accomplishment of the meeting. Without their helps and supports, the preparation for the conference would deem impossible to complete. Finally, I would like to thank all participants for joining the conference. I do hope all participants will have opportunity to explore Bali and enjoy staying in the island of Gods.
Sincerely yours,

Dr. Suhartono
Chairman

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09.45 - 10.30 : Keynote session 2 by Prof. Virasakdi Chongsuvivatwong (Thailand)
10.30 - 11.00 : Break
11.00 - 11.45 : Keynote session 3 by Prof. Dato Dr. Rosihan M. Ali (Malaysia)
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- Dr. Darmaji (Class PM)
- Prof. Dr. Md. Azizul Baten (Class AM)
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- Dr.rar.net Heri Kuswanto (Class SB)

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Wednesday, 21 November 2012

Bali City Tour

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A Study Permutation Theory and Its Application to Enumeration of Latin Square-X

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Abstract. A Latin square of order n is an array or matrix of size $n \times n$ where in each row and column contains n different numbers or symbols. Enumeration of Latin square is not easy problem even using a computer. Until now, the exact number of Latin square is known only for $1 \leq n \leq 11$. The basic in Latin square is permuted n numbers. By considering the characteristic of permutation that appears in Latin square-x, in this paper we will discuss about theory of permutation about Latin square and then applying its to enumeration of Latin square-x.

Keyword:Latin square, Enumeration, Latin square-x, Permutation.

1 Introduction

Enumeration is a problem that associated with counting. In combinatorics, enumeration means determining the exact number of elements of finite sets. For example, established basket ball team consist of 5 players from 7 candidate players. The number teams that can be established are 21.

Latin square of order n is an array or matrix size $n \times n$ with n symbols such that in each row and column filled by the permutation of symbols [4], in other word the entries in each row and in each column are distinct [5]. Latin square firstly introduce by Swiss mathematician, Leonhard Euler. The study of Latin square has long tradition in combinatorics [6]. A Latin square of order n can be called by Latin square-x (doubly diagonalized) if both its diagonals consist of n distinct symbols [3]. An example of Latin square-x is shown in Fig. 1.

Enumeration of Latin square is not easy problem even using computer. Until now, the exact number of Latin square is known only for order $1 \leq n \leq 11$[1, 2, 4].

2	3	1	4
1	4	2	3
4	1	3	2
3	2	4	1

Fig. 1. Latin Square-X of Order 4

The notion of permutation is related to the act of rearranging objects or values. A permutation of a set of objects is an arrangement of those objects into a particular order. For example there are six permutations from element of the set $\{1, 2, 3\}$, that is (1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2) and (3,2,1). For simply, we write a permutation without parentheses and commas. So we will write 123 rather than (1, 2, 3).

In algebra, especially group theory, permutation is a bijective mapping on set X. A family of all permutations from X is called by symmetry group S_X [8], if $X = \{1, 2, 3 \ldots, n\}$ we write S_n rather than S_X. Let $i_1 i_2 \ldots i_n$ be a permutation from $X = \{1, 2, \ldots, n\}$ and defines a function $\alpha : X \to X$ as $\alpha(1) = i_1, \alpha(2) = i_2, \ldots, \alpha(n) = i_n$. For $i \in \{1, 2, \ldots, n\}$ such that $\alpha(i) = i$, then i fixed by α.

Permutation matrix is a square matrix such that in each column and row there is exact one entry 1 and the others is 0. The example of permutation matrix is shown in Fig. 2.

$$A = \begin{bmatrix} 0\,1\,0\,0 \\ 0\,0\,0\,1 \\ 1\,0\,0\,0 \\ 0\,0\,1\,0 \end{bmatrix}$$

Fig. 2. Permutation Matrix of Order 4

Let a_{ij} be entry of permutation matrix at i^{th} row and j^{th} column, from matrix A we can get a permutation function α. If $a_{ij} = 1$, then $\alpha(i) = j$. From A we get $a_{12} = a_{24} = a_{31} = a_{43} = 1$, then we have $\alpha(1) = 2, \alpha(2) = 4, \alpha(3) = 1$, and $\alpha(4) = 3$ and permutation representation of matrix A is 2413 (see Fig. 2).

From a Latin square order n, we can get n permutation matrix that represent fixed number $i \in \{1, 2, \ldots, n\}$. Let L be matrix from Latin square in Fig. 1 (see Fig. 3), then from L we get four permutation matrices, that is L_1, L_2, L_3 and L_4 (see Fig. 4).

$$L = \begin{bmatrix} 2\,3\,1\,4 \\ 1\,4\,2\,3 \\ 4\,1\,3\,2 \\ 3\,2\,4\,1 \end{bmatrix}$$

Fig. 3. Matrix Representation from Latin Square

We can easily check that $L = L_1 + 2L_2 + 3L_3 + 4L_4$, or generally for order n, $L = L_1 + 2L_2 + \; + nL_n$. Let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be permutation of $L_1, L_2, \ldots, L_n$ respectively, again we can check that

$$\alpha_i(k) \neq \alpha_j(k) \text{ for } i, j \in \{1, 2, \ldots, n\} \text{ and } i \neq j \tag{1}$$

$$L_1 = \begin{bmatrix} 0\,0\,1\,0 \\ 1\,0\,0\,0 \\ 0\,1\,0\,0 \\ 0\,0\,0\,1 \end{bmatrix} \qquad L_2 = \begin{bmatrix} 1\,0\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,1 \\ 0\,1\,0\,0 \end{bmatrix}$$

3124 1342

$$L_3 = \begin{bmatrix} 0\,1\,0\,0 \\ 0\,0\,0\,1 \\ 0\,0\,1\,0 \\ 1\,0\,0\,0 \end{bmatrix} \qquad L_4 = \begin{bmatrix} 0\,0\,0\,1 \\ 0\,1\,0\,0 \\ 1\,0\,0\,1 \\ 0\,0\,1\,0 \end{bmatrix}$$

2341 4213

Fig. 4. Permutations matrix from Latin Square and its permutation representation

2 Latin Square Transformation

From a Latin square L, we can get another Latin square L' by the rule below:

- From Latin square L of order n we get n different permutation matrices, that is L_1, L_2,. . ., L_n.
- From n permutation matrices we get n different permutation representation
- The i^{th} row of L' will be filled by permutation representation of matrix L_i for $i = 1, 2, \ldots, n$.

For example, the permutation representation of L_1, L_2, L_3 and L_4 from Fig. 4 is 3124, 1342, 2431 and 4213 respectively, and then these permutations will be filled to 4 x 4 array and get another Latin square, of course uniquely.

$$\begin{array}{|c|c|c|c|} \hline 2&3&1&4 \\ \hline 1&4&2&3 \\ \hline 4&1&3&2 \\ \hline 3&2&4&1 \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{|c|c|c|c|} \hline 3&1&2&4 \\ \hline 1&3&4&2 \\ \hline 2&4&3&1 \\ \hline 4&2&1&3 \\ \hline \end{array}$$

Fig. 5. Latin Square Transformation

It can be checked that after Latin square transformation thrice, we get initial Latin square.

3 Method

3.1 Permutations from Latin Square-X

From number $1, 2, \ldots, n$ in a Latin square we can get n different permutation matrices of order n. Let a_{ij} be entry of permutation matrix at i^{th} row and j^{th} column, then main diagonal entry is a_{ii} and secondary diagonal entry is a_ij with $i + j = n + 1$.

We know that in permutation matrix, if $a_{ij} = 1$, then a permutation function α satisfy $\alpha(i) = j$. Because the entries at main diagonal and secondary diagonal are distinct, we can conclude that permutation from Latin square-x i.e α, satisfy:

i. There is exactly one i such that $\alpha(i) = i$.
ii. There is exactly one j such that $\alpha(j) + j = n + 1$. (2)

From (2) i we know that there is exactly one $i \in \{1, 2, \ldots, n\}$ that fixed by α.

3.2 Partition of Permutation

Let P_n be a set all permutation from Latin square-x of order n. P_n will be partitioned into subsets namely $P_{n,i:j}$ based on i, j that satisfy (2). For example $1423 \in P_{4,1:3}$ because $\alpha(1) = 1, \alpha(3) + 3 = 5$ and there is no other $i, j \in \{1, 2, 3, 4\}$ satisfy (2).

For n even, P_n can be partitioned into $n(n-2)$ subsets and for n odd, P_n can be partitioned into $(n-1)^2$ subsets. For example

- $P_4 = \{P_{4,1:2}, P_{4,1:3}, P_{4,2:1}, P_{4,2:4}, P_{4,3:1}, P_{4,3:4}, P_{4,4:2}, P_{4,4:3}\}$.
- $P5 = \{P_{5,1:2}, P_{5,1:4}, P_{5,2:1}, P_{5,2:5}, P_{5,3:3}, P_{4,4:1}, P_{5,4:5}, P_{5,5:2}, P_{5,5:4}\}$.

3.3 Enumeration of Permutation

We know that all permutation in P_n satisfy (2). Because of partitioning of P_n into $P_{n,i:j}$, to enumerate all permutations from Latin square is equal to enumerate the cardinality of each $P_{n,i:j}$ or $|P_{n,i:j}|$. For n even and n odd with $i \neq j$, it is easy to check that the value of $|P_{n,i:j}|$ is not depend i and j, so $|P_{n,i:j}|$ is the same for all i, j. For n odd and $i = j$, the value of $|P_{n,i:i}|$ is higher than others. The value of $|P_n|$ is equal to the sum all $|P_{n,i:j}|$. To enumerate all permutation in P_n is used algebra software GAP (*Group*, *Algorithm*, *Programming*) [9].

For example,

- $P_4 = \{1342, 1423, 4213, 3241, 4132, 2431, 2314, 3124\}$
- $P_5 = \{14253, 14532, 13524, 15423, 52134, 52413, 32451, 42531, 21354, 25314, 41352, 45312, 51243, 53142, 23541, 35241, 24135, 34215, 31425, 43125\}$

3.4 Enumeration of Latin Square-X

It has been explained that by Latin square transformation, we get other Latin square uniquely and all permutations from Latin square-x satisfy (1). Because the uniquely of Latin square from Latin square transformation, the enumeration problem of all possible Latin square-x is equal to problem of counting "how to select and arrange" n permutations from P_n to $n \times n$ array that produce Latin square. This problem transformation made difficulty level of enumeration of Latin square-x decrease.

Because P_n has been partitioned into $P_{n,i:j}$, we need to consider value of i, j such that the n permutations that we selected can produce Latin square.

Of course we cant select two permutations from the same $P_{n,i:j}$, because the number at position i, j is the same. For example, both 14253 and 14532 are permutation from $P_{5,1:2}$, the number at 1st and 2nd position are the same, that is 1 and 4 respectively. Then from n permutations, the index i, j each $P_{n,i:j}$ are all different.

We defined *collection of permutation* Kn that can produce Latin square. For example,

$$K4 = \{\{P_{4,1:2}, P_{4,2:1}, P_{4,3:4}, P_{4,4:3}\}, \{P_{4,1:2}, P_{4,2:4}, P_{4,3:1}, P_{4,4:3}\}, \{P_{4,1:3}, P_{4,2:1}, P_{4,3:4}, P_{4,4:2}\}, \{P_{4,1:3}, P_{4,2:4}, P_{4,3:1}, P_{4,4:2}\}\}.$$

Then we have four possible collection of permutation for order 4, or we can write $|K4| = 4$. Using GAP, we get $|K5| = 4, |K6| = |K7| = 8, |K8| = |K9| = 4752$. We will enumerate the number of Latin square-x manually for order 4, for order 5, 6 and 7 we enumerate using algebra software GAP.

For order 4 we have $P_4 = \{1342, 1423, 4213, 3241, 4132, 2431, 2314, 3124\}$ and $K4 = \{\{P_{4,1:2}, P_{4,2:1}, P_{4,3:4}, P_{4,4:3}\}, \{P_{4,1:2}, P_{4,2:4}, P_{4,3:1}, P_{4,4:3}\}, \{P_{4,1:3}, P_{4,2:1}, P_{4,3:4}, P_{4,4:2}\}, \{P_{4,1:3}, P_{4,2:4}, P_{4,3:1}, P_{4,4:2}\}\}$. Hence, we have four possible.

i. First collection
First collection is $\{P_{4,1:2}, P_{4,2:1}, P_{4,3:4}, P_{4,4:3}\} = \{1342, 4213, 2431, 3124\}$. By this collection we can produce a Latin square.

$$\begin{matrix}1342\\4213\\2431\\3124\end{matrix} \rightarrow \begin{array}{|c|c|c|c|}\hline 1&3&4&2\\\hline 4&2&1&3\\\hline 2&4&3&1\\\hline 3&1&2&4\\\hline\end{array} \rightarrow \text{Latin Square}$$

ii. Second collection
Second collection is $\{P_{4,1:2}, P_{4,2:4}, P_{4,3:1}, P_{4,4:3}\} = \{1342, 3241, 4132, 3124\}$. By this collection we cant produce a Latin square.

$$\begin{matrix}1342\\3241\\4132\\3124\end{matrix} \rightarrow \begin{array}{|c|c|c|c|}\hline 1&3&4&2\\\hline 3&2&4&1\\\hline 4&1&3&2\\\hline 3&1&2&4\\\hline\end{array} \rightarrow \text{not Latin Square}$$

iii. Third collection
Third collection is $\{P_{4,1:3}, P_{4,2:1}, P_{4,3:4}, P_{4,4:2}\} = \{1423, 4213, 2431, 2314\}$. By this collection we cant produce a Latin square.

$$\begin{matrix}1423\\4213\\2431\\2314\end{matrix} \rightarrow \begin{array}{|c|c|c|c|}\hline 1&4&2&3\\\hline 4&2&1&3\\\hline 2&4&3&1\\\hline 2&3&1&4\\\hline\end{array} \rightarrow \text{not Latin Square}$$

iv. Fourth collection
Fourth collection is $\{P_{4,1:3}, P_{4,2:4}, P_{4,3:1}, P_{4,4:2}\} = \{1423, 3241, 4132, 2314\}$. By this collection we can produce a Latin square.

$$\begin{matrix} 1423 \\ 3241 \\ 4132 \\ 2314 \end{matrix} \rightarrow \begin{array}{|c|c|c|c|} \hline 1&4&2&3 \\ \hline 3&2&4&1 \\ \hline 4&1&3&2 \\ \hline 2&3&1&4 \\ \hline \end{array} \rightarrow \text{not Latin Square}$$

From four possible collections, we get a Latin square only from the first and the fourth. Then for $n = 4$, the total number of Latin square-x is $2 \times 4! = 48$, because we can arrange rows in 4! ways.

4 Result and Discussion

We know that all permutations from Latin square satisfy (2). The complete result of enumeration of P_n is shown in Table 1. After enumeration of P_n, we can enumerate the number of Latin square-x by chose and arrange n permutations to $n \times n$ array that produce a Latin square, the complete result of enumeration of Latin square-x is shown in Table 2.

Table 1. The number of permutation from Latin square-x [7]

P_n	$\lvert P_{n,i:j}\rvert^*$	$\lvert P_{n,i:j}\rvert^{**}$	$\lvert P_n\rvert$
P_4	1	0	$1 \times 8 = 8$
P_5	2	2	$2 \times 8 + 4 = 20$
P_6	4	0	$4 \times 24 = 96$
P_7	24	80	$24 \times 24 + 80 = 656$
P_8	116	0	$116 \times 48 = 5568$
P_9	920	4752	$920 \times 48 + 4752 = 48912$

$*$: for $i \neq j$,$**$: for $i = j$

Table 2. The number of Latin square-x [7]

n	$L(n)$	Total
4	2	$2 \times 4! = 48$
5	8	$8 \times 5! = 960$
6	128	$128 \times 6! = 92160$
7	171200	$171200 \times 7! = 862848000$

5 Conclusion

After analyzed the permutation from Latin square-x, it can be concluded that all permutation satisfy (2), then using those permutations we can enumerate the number of Latin square-x by chose and arrange n different permutations to $n \times n$ array. If from that we get a Latin square then we have Latin square-x.

6 Acknowledgements

This paper based on my bachelors final project. Thank you to Dr. A. F. Jarvis for helpful discussion and suggestion although via e-mail.

References

1. Felgenhauer, B., Jarvis, A.F., "Mathematics of Sudoku I", *Mathematics Spectrum* vol 39, pp 315-333 (2005)
2. Felgenhauer, B., Jarvis, A.F., "Enumerating Possible Sudoku Grids", http://www.afjarvis.staff.shef.ac.uk/sudoku/sudoku.pdf, 2005.
3. Gergely, E., "A Simple Method for Constructing Doubly Diagonalized Latin Squares", *Journal of Combinatorial Theory* (A), vol. 16, pp 266-272, 1975.
4. Hulpke, A., Kaski, P., Östergård, P.R.J., "The Number of Latin Square of Order 11", *Mathematics of Computation* vol. 80, number 274, pp. 1197-1219, 2010.
5. McKay, B.D., Wanless, I.M., "On the Number of Latin Squares", *Ann. Combinatorics* vol 9, pp 334-344, 2005.
6. Mészáros, K., "Generalized Latin Square and Their Defining Set", *Discrete Mathematics* vol. 308, pp. 2366-2378, 2007.
7. Mufid, M.S., 2012, "A study permutation theory and its application to enumeration of Latin Square-X and Sudoku-X" (in Indonesian), *Bachelor Final Project*, Mathematics and Science Faculty, Institut Teknologi Sepuluh Nopember, Surabaya.
8. Rotman, J.J., 2003, *Advance Modern Algebra*, Prentice Hall, New Jersey.
9. The GAP Group, GAP – Group, Algorithm, and Programming, Version 4.5.6; 2012. (http://www.gap-system.org)