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The International Conference on Numerical Analysis & Optimization

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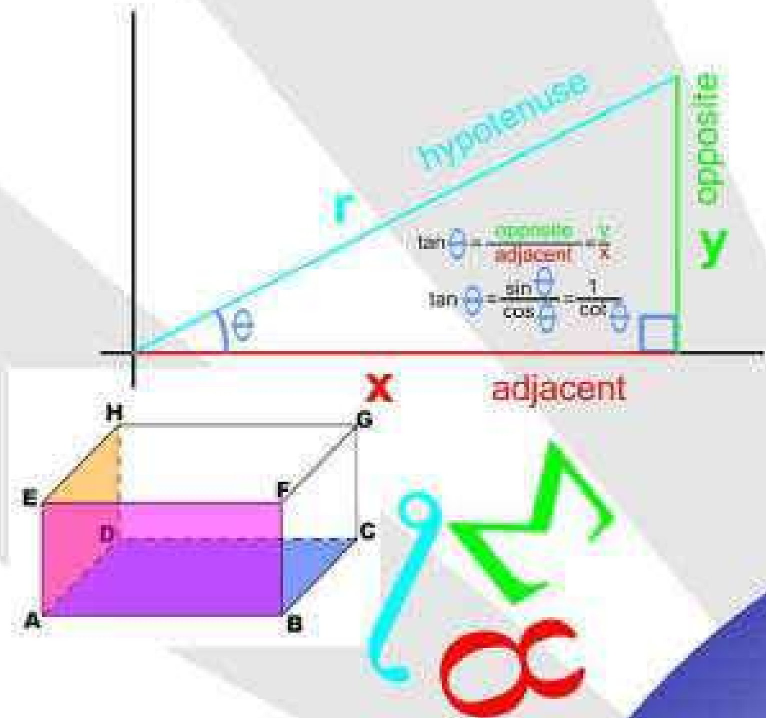

June 6 - 8, 2011

Hosted by :

Departement of Mathematics

Faculty of Mathematics and Natural Sciences

The International Conference on Numerical Analysis and Optimization (ICeMATH2011)

The field of numerical analysis predates the invention of modern computers by many centuries. Numerical analysis is the area of mathematics and computer science that creates, analyzes, and implements algorithms for solving numerically the problems of continuous mathematics. Such problems originate generally from real-world applications of algebra, geometry, and calculus, and they involve variables which vary continuously.

On the other hand, Numerical Optimization is defined as a scientific approach in finding the finest solution of a particular problem that is interpreted in mathematical models. Hence, the combination of numerical analysis with numerical optimization is highly important for scientific efforts in the areas of developmental work as well as humanity in general.

Therefore, on the occasion of the 50th anniversary of its founding celebration, Universitas Ahmad Dahlan (UAD) with the collaboration of Journal KALAM has initiated **The International Conference on Numerical Analysis and Optimization (ICeMATH 2011)** to be held at Yogyakarta, Indonesia.

Objectives:

- Provide a platform for researchers, professionals, and academicians to exchange ideas and discuss their research findings.
- Encourage future collaborations between participants.
- Provide room for researchers to discuss their thoughts and views on the development of this field that can contribute towards future works as well as being a very beneficial program for all participants.

Topic of Discussions:

Numerical Analysis, Numerical Methods, Operations Research, Mathematics, Statistics, Numerical Optimization, Differential Equation, Applied Mathematics and Statistics, Interval Mathematics, Fuzzy, Computational Mathematics, Combinatory, Algebra, Engineering Mathematics, Mathematics Education

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6^{TH} – 8^{TH} JUNE 2011

UNIVERSITAS AHMAD DAHLAN, YOGYAKARTA, INDONESIA

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Part VIII - Statistics

MODELLING OF ELECTRICAL TRAIN (ET) NETWORK SYSTEM USING MAX-PLUS ALGEBRA

Siti Alfiah, Subiono

Abstract. Transportation is important thing for society to do their daily mobilities. In the city society which has high mobility characteristics, like Jakarta, there are many complex problems in transportation. One of the problem is traffic jam. In order to reduce the traffic jam in Jakarta, the *electrical train* (ET) KA *Commuter* Jabodetabek is built. As a cheap and a fast transportation facility, ET is widely used by society. One of important issues related to ET network system is often occurrence of inaccuracies arriving and departure times of trains at each station, so it is complained by society. This research is purposed to modelling ET network system using max-plus algebra, which the duration of driving time between station is given as an interval of times with elements in max-plus algebra. This ET network system can be formed into adjacency matrix with element in the form of interval in max-plus algebra.

Keywords and Phrases : *Max-Plus Algebra, Model of ET Network System.*

1. INTRODUCTION

Discussion about how repairing the public transportation system must be improved considerably. Related to this problem, the theory of max-plus algebra is one of theory that can used to modelling, analysis and control of transportation network system. In [6] had used max-plus algebra to scheduling bus line network in the city with case-study of TransJakarta bus network. The model is constructed based on the number of lines that have been active operated, the maximum numbers of busses that allocated and the rule of synchronization for each line. From this research, the design of periodic schedule of departure time is obtained. In [1] had discussed about max-plus algebra approach to transportation system, especially railway system. A scheduled railway system can efficiently be modelled as a discrete event dynamic system (DEDS) using max-plus algebra. There are 11 lines of intercity Dutch railway system is modelled here. In [4] had discussed about modelling whole connection of trains in Dutch railway system. The constructed model more complex than [1]. In [2] had discussed about modelling scheduled railway system and analysis stability and realizability timetable using max-plus algebra. In this paper max-plus algebra is used to modelling KRL network system with timetable. The driving times and the timetable are given by interval form in $R_{\max}$.

2. MAX-PLUS ALGEBRA

The max-plus algebra is defined by the operations addition and maximization applied to the real numbers, extended with minus infinity.

Definition *The max-plus algebra* ($\mathrm{R}_{\max}$, $\oplus$, $\otimes$) is defined as follows [1]:

(a) $\mathrm{R}_{\max} \overset{def}{=} \mathrm{R}\cup\{-\infty\}$, *where* R *is the set of real numbers*;

(b) $\oplus$ *is maximization in the usual ordering of* $\mathrm{R}_{\max}$;

(c) $\otimes$ *is the usual addition, where*

$$a \otimes -\infty \overset{def}{=} -\infty \otimes a \overset{def}{=} -\infty \text{ for all } a \in \mathrm{R}_{\max}.$$

Max-plus algebra having 0 as neutral element with respect to $\otimes$, will be denoted by e and $-\infty$ as neutral element with respect to $\oplus$ and absorbing element for $\otimes$, will be denoted by ε.

3. INTERVAL MAX-PLUS ALGEBRA

Interval max-plus algebra is extention of max-plus algebra [3].

Definition *Interval max-plus algebra is defined as* $\mathrm{I(R)}_{\max} = \{x = [\underline{x}, \overline{x}] \mid \underline{x}, \overline{x} \in R, \varepsilon \prec_m \underline{x} \preceq_m \overline{x}\} \cup \{[\varepsilon, \varepsilon]\}$.

In $\mathrm{I(R)}_{\max}$ operations $\oplus$ and $\otimes$ are defined as:

$x \oplus y = [\underline{x} \oplus \underline{y}, \overline{x} \oplus \overline{y}]$ and $x \otimes y = [\underline{x} \otimes \underline{y}, \overline{x} \otimes \overline{y}]$, $\forall x, y \in \mathrm{I(R)}_{\max}$

Example

$[-1,1] \oplus [1,3] = [1,3]$ and $[-1,1] \otimes [1,3] = [0,4]$.

Definition *Let* $\mathrm{I(R)}_{\max}^{n\times m}$ *be the set of n x m matrices in interval max-plus algebra. The interval matrix is set of matrices that have interval value and be written as* $\mathbf{A} = (\underline{A}, \overline{A})$, *where* $\underline{A}, \overline{A} \in \mathrm{R}_{\max}^{nxm}$ *and* $\underline{A} \le \overline{A}$.

Example

Let $\mathbf{A} = \begin{pmatrix} [-1,3] & [0,2] & [6,8] \\ [\varepsilon,\varepsilon] & [3,5] & [2,5] \end{pmatrix}$, so $\underline{A} = \begin{pmatrix} -1 & 0 & 6 \\ \varepsilon & 3 & 2 \end{pmatrix}$ and $\overline{A} = \begin{pmatrix} 3 & 2 & 8 \\ \varepsilon & 5 & 5 \end{pmatrix}$

Operation $\oplus$ and $\otimes$ in $\mathrm{I(R)}_{\max}$ can be extended for operation-operation matrix in interval max-plus algebra.

4. MODELLING OF RAILWAY SYSTEM

4.1 Desired Model

A train network can be modelled as a system of the form [1]:

$$\left.\begin{aligned} x(k+1) &= Ax(k) \oplus d(k+1) \\ y(k) &= Cx(k) \\ x(0) &= x_0 \end{aligned}\right\} \qquad (1)$$

The vector $x(k)$ contains the k[th] departure times of all trains including auxiliary ones. Vector

$x(k) = [x_1(k), ..., x_n(k), x_{n+1}(k), ..., x_{n+r}(k)]'$, with 1, 2, …, n referring to the real trains, and n+1, …, $n + r$ related to auxiliary variables, to be interpreted as dummy trains. The vector $d(k+1)$ is the timetable for the k+1st departures. The initial state is $x(0)$, although the 0th departure of a train does not have a clear interpretation. Since we only can look at what happens with x_1,…, x_n, the output matrix is $C = [e_n \quad \varepsilon_{nxr}]$. So the output $y(k)$ is the observation of the departure times of the actual trains.

4.2 Timetable

The vector $d(k) \epsilon \boldsymbol{R}_{max}^n$ contains the scheduled k^{th} departure times for all trains. Because trains are scheduled modulo T, we obtain

$d(k) = d(0)T^k$

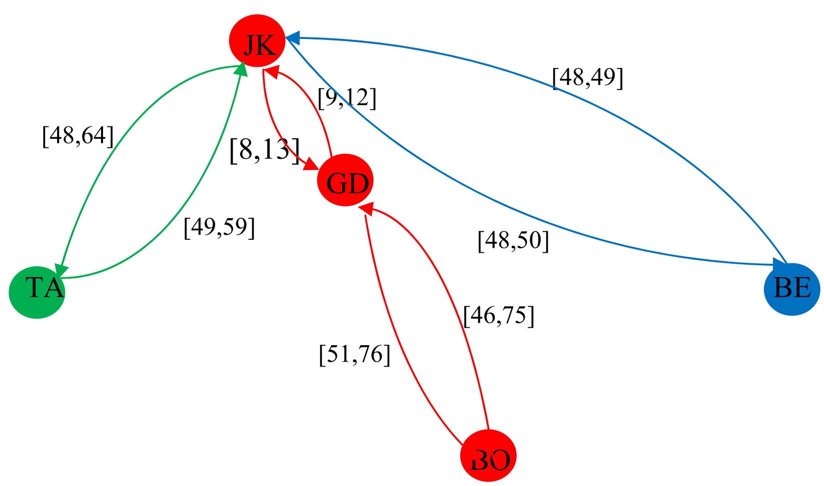
holds for all k. This can also be written as

$d(k) = d(0) + (k.T) \otimes \eta$ where $\eta = [e,e,\ldots,e]' \epsilon \boldsymbol{R}_{max}^n$, the column vector that all the element are e in max-plus algebra.

5. MODELLING OF ELECTRICAL TRAIN (ET) NETWORK SYSTEM

5.1 KA Commuter Jabodetabek Network System

The railroad KA Commuter Jabodetabek network system is taken from [8]. In this paper will be modelled the railroad from Jakarta Kota to Bogor, Jakarta Kota to Bekasi and Jakarta Kota to Tangerang with platform are Jakarta Kota, Gondangdia, Bogor, Bekasi and Tangerang. The data about a fixed number of trains on each line and the driving times of the trains from one station to the next are taken from [7] with respect to a normal working day from 04:00 pm until 07:00 pm. These data are used to derive the driving times (the difference between the arriving and departure time of the trains from one station to the next). Here assumed that the cyclicity of the timetable is 180 minutes. Distribution number of trains that operating on each line is determined using reference time 05:15 pm. The directed graph of KA Commuter Jabodetabek as follow:



Graph KA Commuter Jabodetabek Network

JK – BO : Line 1	JK : Jakarta Kota	BO : Bogor
JK – BE : Line 2	GD : Gondangdia	BE : Bekasi
JK – TA : Line 3	TA : Tangerang	

5.2 Physical Specification

The physical specification is defined as follow:

i) The lines in KA Commuter Jabodetabek network

ii) The number and distribution of trains on each line

iii) The synchronization rules between the trains

There are three lines that modelled here. The driving times of trains and the timetable of departure times are given below:

Line	Departure Station	Stop Station	Timetable	Driving time (minutes)	The number of trains
1	Jakarta Kota	Gondangdia	5 – 179	8 – 13	1
1	Gondangdia	Bogor	5 – 165	51 – 76	3
1	Bogor	Gondangdia	20 – 180	46 – 75	5
1	Gondangdia	Jakarta Kota	8 – 178	9 – 12	2
2	Jakarta Kota	Bekasi	40 – 160	48 – 50	2
2	Bekasi	Jakarta Kota	38 – 180	48 – 49	1
3	Jakarta Kota	Tangerang	0 – 165	48 – 64	1
3	Tangerang	Jakarta Kota	60 – 95	49 – 59	1

The synchronization rules are given as follows:

- On line 1, the trains that depart for the (k + 1)-st time from Jakarta Kota to Gondangdia should wait for the k-th arrival of trains which departed from Bekasi to Jakarta Kota and wait for the k-th arrival of trains which departed from Tangerang to Jakarta Kota.
- On line 2, the trains that depart for the (k + 1)-st time from Jakarta Kota to Bekasi should wait for the (k-1)-th arrival of trains which departed from Gondangdia to Jakarta Kota and wait for the k-th arrival of trains which departed from Tangerang to Jakarta Kota.
- On line 3, the trains that depart for the (k + 1)-st time from Jakarta Kota to Tangerang should wait for the (k-1)-th arrival of trains which departed from Gondangdia to Jakarta Kota and wait for the k-th arrival of trains which departed from Bekasi to Jakarta Kota.

5.3 Model of KA Commuter Jabodetabek Network System

Before desire model, needed definition variable that will be used in system. Vector $x(k)$ contains departure times for the k-th of trains in each station as follows:

Line	Variable	Departure Station	Stop Station
1	x_1	Jakarta Kota	Gondangdia
1	x_2	Gondangdia	Bogor
1	x_3	Bogor	Gondangdia
1	x_4	Gondangdia	Jakarta Kota

2	x_5	Jakarta Kota	Bekasi
2	x_6	Bekasi	Jakarta Kota
3	x_7	Jakarta Kota	Tangerang
3	x_8	Tangerang	Jakarta Kota

The model of network system before synchronization as follows:

i) Line 1

$$\begin{aligned} x_1(k+1) &= [9,12] \otimes x_4(k-1) \oplus d_1(k+1) \\ x_2(k+1) &= [8,13] \otimes x_1(k) \oplus d_2(k+1) \\ x_3(k+1) &= [51,76] \otimes x_2(k-2) \oplus d_3(k+1) \\ x_4(k+1) &= [46,75] \otimes x_3(k-4) \oplus d_4(k+1) \end{aligned} \tag{2}$$

ii) Line 2

$$\begin{aligned} x_5(k+1) &= [48,49] \otimes x_6(k) \oplus d_5(k+1) \\ x_6(k+1) &= [48,50] \otimes x_5(k-1) \oplus d_6(k+1) \end{aligned} \tag{3}$$

iii) Line 3

$$\begin{aligned} x_7(k+1) &= [49,59] \otimes x_8(k) \oplus d_7(k+1) \\ x_8(k+1) &= [48,64] \otimes x_7(k) \oplus d_8(k+1) \end{aligned} \tag{4}$$

Based on synchronization rules in 5.2, can be constructed model of whole railway system as follows:

i) Line 1

$$\left.\begin{aligned} x_1(k+1) &= ([9,12] \otimes x_4(k-1)) \oplus ([48,49] \otimes x_6(k)) \\ &\quad \oplus ([49,59] \otimes x_8(k)) \oplus d_1(k+1) \\ x_2(k+1) &= [8,13] \otimes x_1(k) \oplus d_2(k+1) \\ x_3(k+1) &= [51,76] \otimes x_2(k-2) \oplus d_3(k+1) \\ x_4(k+1) &= [46,75] \otimes x_3(k-4) \oplus d_4(k+1) \end{aligned}\right\} \tag{5}$$

ii) Line 2

$$\left.\begin{aligned} x_5(k+1) &= ([48,49] \otimes x_6(k)) \oplus ([9,12] \otimes x_4(k-1)) \\ &\quad \oplus ([49,59] \otimes x_8(k)) \oplus d_5(k+1) \\ x_6(k+1) &= [48,50] \otimes x_5(k-1) \oplus d_6(k+1) \end{aligned}\right\} \tag{6}$$

iii) Line 3

$$\left.\begin{aligned} x_7(k+1) &= ([49,59] \otimes x_8(k)) \oplus ([9,12] \otimes x_4(k-1)) \\ &\quad \oplus ([48,49] \otimes x_6(k)) \oplus d_7(k+1) \\ x_8(k+1) &= [48,64] \otimes x_7(k) \oplus d_8(k+1) \end{aligned}\right\} \tag{7}$$

Then, model (5), (6) and (7) above can be stated in standard form max-plus algebra model as follows:

$$x(k+1) = \bigoplus_{p=1}^{M} \left(A_p \otimes x(k+1-p)\right) \oplus d(k+1)$$

There are 5 matrices A_p with $p = \{1, 2, 3, 4, 5\}$ and size of each matrix 8 x 8.
Matrices A_1, A_2, A_3, A_4 and A_5 as follows:

$$A_1 = \begin{pmatrix} [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [48,49] & [\varepsilon,\varepsilon] & [49,59] \\ [8,13] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [48,49] & [\varepsilon,\varepsilon] & [49,59] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [48,49] & [\varepsilon,\varepsilon] & [49,59] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [48,64] & [\varepsilon,\varepsilon] \end{pmatrix}$$

$$A_2 = \begin{pmatrix} [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [9,12] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [9,12] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [48,50] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [9,12] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \end{pmatrix}$$

$$A_3 = \begin{pmatrix} [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [51,76] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \end{pmatrix}$$

$$A_4 = \begin{pmatrix} [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \end{pmatrix}$$

$$A_5 = \begin{pmatrix} [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [46,75] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \\ [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] & [\varepsilon,\varepsilon] \end{pmatrix}$$

Therefore model (5), (6) and (7) can be written as:

- $$x(k+1) = A_1 x(k) \oplus A_2\, x(k-1) \oplus A_3\, x(k-2) \oplus A_4\, x(k-3) \oplus A_5 x(k-4) \oplus d(k+1)$$

6. CONCLUSIONS

Max-plus algebra can be used to constructed model of electrical train (ET) network system. The model can be written as $x(k+1) = \bigoplus_{p=1}^{M} \left(A_p \otimes x(k+1-p)\right) \oplus d(k+1)$

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