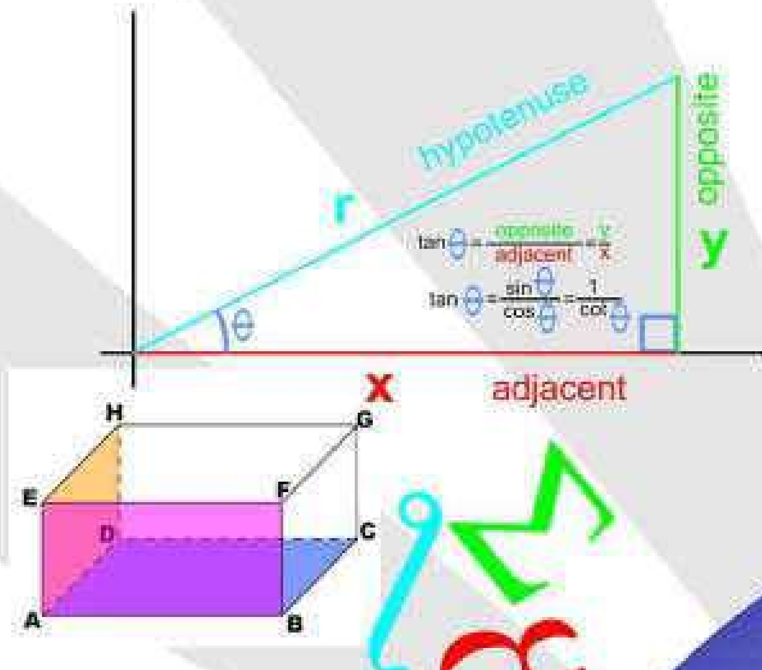
Proceeding ICeMATH 2011

The International Conference on Numerical Analysis & Optimization

No. ISBN : 978-602-98919-1-1



June 6 - 8, 2011

Hosted by :
Departement of Mathematics
Faculty of Mathematics and Natural Sciences

The International Conference on Numerical Analysis and Optimization (ICeMATH2011)

The field of numerical analysis predates the invention of modern computers by many centuries. Numerical analysis is the area of mathematics and computer science that creates, analyzes, and implements algorithms for solving numerically the problems of continuous mathematics. Such problems originate generally from real-world applications of algebra, geometry, and calculus, and they involve variables which vary continuously.

On the other hand, Numerical Optimization is defined as a scientific approach in finding the finest solution of a particular problem that is interpreted in mathematical models. Hence, the combination of numerical analysis with numerical optimization is highly important for scientific efforts in the areas of developmental work as well as humanity in general.

Therefore, on the occasion of the 50th anniversary of its founding celebration, Universitas Ahmad Dahlan (UAD) with the collaboration of Journal KALAM has initiated **The International Conference on Numerical Analysis and Optimization (ICeMATH 2011)** to be held at Yogyakarta, Indonesia.

Objectives:

- Provide a platform for researchers, professionals, and academicians to exchange ideas and discuss their research findings.
- Encourage future collaborations between participants.
- Provide room for researchers to discuss their thoughts and views on the development of this field that can contribute towards future works as well as being a very beneficial program for all participants.

Topic of Discussions:

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Part VIII - Statistics

COMPUTATION DECOMPOSITION HAAR WAVELET BASED MAX-PLUS ALGEBRA

Hergian Dinarina[1], Mahmud Yunus[2], Subiono[3]

Abstract. In this paper, the formula a wavelet transform max-plus is proposed, which is a wavelet transform to compute coefficients of decomposition in max-plus algebra system. This wavelet transform is proposed by simplifying the morphological wavelet transform. It is a very reasonable thing to do because in max-plus algebra system, where analysis and synthesis operation are defined by max and standard sum, so that, can minimize a complexity in a calculation, therefore, they are can ensure a data information. The operators that used in max-plus algebra system are closely related to operators in mathematical morphology, such as dilation and erosion, used to formulate the wavelet transform morphology. The formulation of this transformation will be used for decompositioning of a higher-resolution signal into a lower-resolution signals. A recursive wavelet transform, will decompose a signal into an approximation signal (low frequency part) and some detail signal (high frequency part). We will also be development an algorithm for computing coefficients of decomposition in max-plus algebra system.

Keywords and Phrases: *wavelet transform, wavelet morphology, max-plus algebra.*

1. INTRODUCTION

At the morphological Haar wavelet transform, the average operator is replaced by the operator max or min operator (Xiang, et al [8]). The operator max / min is reminiscent of the max-plus algebra. In max-plus algebra, the operator used is the max and sum operators (Subiono [6]). This is one reason that, wavelet transformation can be formulated to max-plus algebra system. Max-plus algebra are able to make the calculation process becomes simpler, because the operator is used to minimize errors that would arise when doing numerical computations with general operators. In the process of signal decomposition, the error appears to result in reduced accuracy of the data so that information contained in the initial signal can be reduced validity.

1.1 The Formula of Max-Plus Wavelet Haar Transformation

In this section, will present the process of formation of the Haar wavelet transformation formula in the max-plus algebra system. This process starts from the decomposition formula using morphological Haar wavelet transform has been written by Heijmans, et al[5]. In the process of decomposition of the data at this stage of analysis, the formula for the approximation of operators, ie operators that are used to obtain the signal approximation, the classical Haar wavelet (Xiang, et al [8]) is often referred to as the operator of averaging replaced by morphological Haar wavelet transform formula in max operator, the following :

$$\Psi_k^{\vee}(l) = \Psi_{k+1}^{\vee}(2l) \vee \Psi_{k+1}^{\vee}(2l+1)\,, 0 \le l \le 2^k - 1 \qquad (1)$$

For service details, no change in the formula, namely :

$$W_{k,l}^{\vee} = \Psi_{k+1}^{\vee}(2l) - \Psi_{k+1}^{\vee}(2l+1), 0 \le l \le 2^k - 1 \qquad (2)$$

1.1.1 Signal analysis proces

The following scheme shows the main signal $V^{(0)}$ decomposed into an approximation signal, which is located at the top of the scheme (Heijmans, et al[5]) and some signal detail, namely the gray starts from the top.

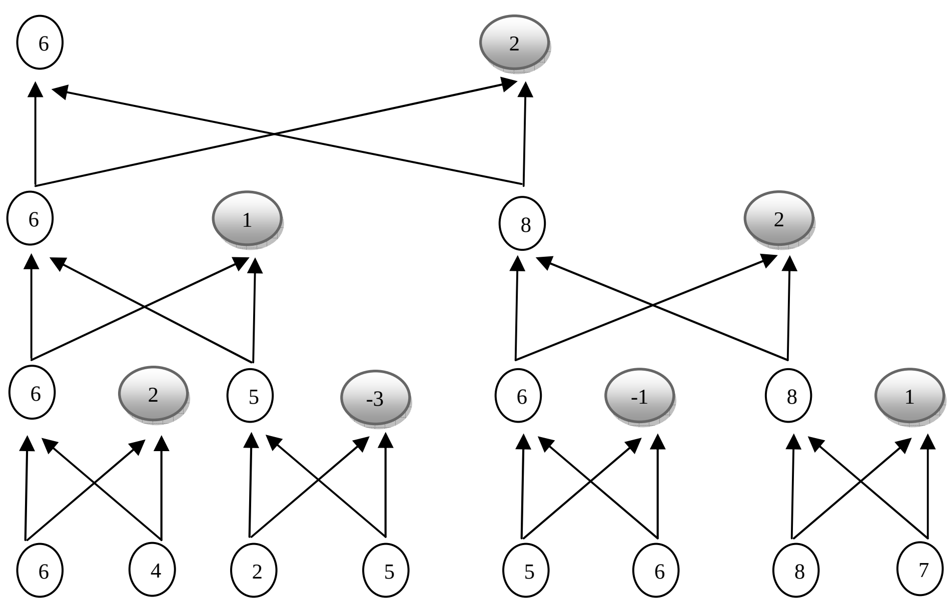

Figure 1. Schematic process of analysis with morphological Haar wavelet transformation.

At the morphological Haar wavelet transform for 1-dimensional (Xiang, et al [8]), all operators in it works in principle the same as the classical Haar wavelet transform, only the average operator in effect on the Haar wavelet transform is replaced by a nonlinear max operator (denoted by $\vee$) or min (denoted by $\wedge$).

Morphological Haar wavelet transform for the analysis are:

$$\Psi_{\mathrm{k}}^{\vee}(l) = \Psi_{\mathrm{k+1}}^{\vee}(2l) \vee \Psi_{\mathrm{k+1}}^{\vee}(2l+1), 0 \leq l \leq 2^k - 1$$

$$\mathrm{W}_{\mathrm{k,l}}^{\vee} = \Psi_{\mathrm{k+1}}^{\vee}(2l) - \Psi_{\mathrm{k+1}}^{\vee}(2l+1), 0 \leq l \leq 2^k - 1$$

As for the transformation of 2-dimensional data decomposition (Heijman, et al, [5]) can also be done by seeing in figure 2. Let $V^{(0)}$ denote the point $n, 2n$, that is $(m,n), (2m, 2n) \in \mathrm{Z}^2$, then the points,, namely the points $2n_+$, $2n^+$, $2n_+^+$, are $(2m+1, 2n)$, $(2m, 2n+1)$, $(2m+1, 2n+1)$. In this case, the rows of the main signal and an approximation signal is all functions from to, where as the signal sequence is a function of the details. Then the approximation operators are defined in Heijman, et al, [5].

In general, the decomposition of 2-dimensional data can be described as follows. Processing of this data can be valid for 2-dimensional $2n$, with $n \geq 1$.

$x(2n)$	$x(2n^+)$
$x(2n_+)$	$x(2n_+^+)$

$\longrightarrow$

Sinyal hampiran	Detail vertikal
Detail horisontal	Detail diagonal

Figure 2. 2-dimensional processing using morphological Haar wavelet transforms

The next stage is to establish a formula for the max-plus wavelet transform for the analysis stage, by replacing the max operator to operator o-plus and plus to be o-cross, while the minus operator can be replaced by the operator o-cross after previously done by the operator plus opponent

- 1-dimensional

Formula approximation operator for 1-dimensional data is:

$$\Psi_{\mathrm{k}}^{\vee}(l) = \Psi_{\mathrm{k+1}}^{\vee}(2l) \oplus \Psi_{\mathrm{k+1}}^{\vee}(2l+1)$$

and formula operator detail :

$$\mathrm{W}_{\mathrm{k,l}}^{\vee} = \Psi_{\mathrm{k+1}}^{\vee}(2l) \otimes \left(-\Psi_{\mathrm{k+1}}^{\vee}(2l+1)\right)$$

- 2-dimensional

While the formula for the operator detail:

$$\Psi^{\uparrow}(x)(n) = x(2n) \oplus x(2n_+) \oplus x(2n^+) \oplus x(2n_+^+)$$

And the formula for the signal operator detail:

$$W_v(x)(n) = \frac{1}{2}\Big(x(2n) \otimes \big(-x(2n^+)\big) \otimes x(2n_+) \otimes \big(-x(2n_+^+)\big)\Big)$$
$$W_h(x)(n) = \frac{1}{2}\Big(x(2n) \otimes \big(-x(2n_+)\big) \otimes x(2n^+) \otimes \big(-x(2n_+^+)\big)\Big)$$
$$W_d(x)(n) = \frac{1}{2}\Big(x(2n) \otimes \big(-x(2n_+)\big) \otimes \big(-x(2n^+)\big) \otimes x(2n_+^+)\Big)$$

After decomposition of the analysis process, will then decompose with the process of synthesis. This process can be referred to as the inverse of the analysis process, where the approximation signal and detail signal sequence is used to recover the main signal. The processing by synthesis process is also started with a 1-dimensional process, to more easily understand, and then continued with 2-dimensional process.

1.1.2 Signal synthesis process

Formula for the synthesis of 1-dimensional and 2-dimensional process using morphological Haar wavelet transform (Heijman, et al [5]).

In the same way on the formula for process analysis, can be obtained morphological Haar wavelet transform formula for the synthesis of which is written in the max-plus algebra operators, namely:

- 1-dimensional :

$$\Psi^{\downarrow}(x)(2n) = \Psi^{\downarrow}(x)(2n+1) = x(n)$$

and,

$$W^{\downarrow}(y)(2n) = -y(n) \oplus 0$$
$$W^{\downarrow}(y)(2n+1) = -(y(n) \oplus 0)$$

also,

$$x(2n) = \Psi^{\downarrow}(x)(2n) \otimes \big(-W^{\downarrow}(y)(2n)\big)$$
$$x(2n+1) = \Psi^{\downarrow}(x)(2n+1) \otimes \big(-W^{\downarrow}(y)(2n+1)\big)$$

- 2-dimensional

As for the 2-dimensional formula is as follows:

$$\Psi^{\downarrow}(x)(2n) = \Psi^{\downarrow}(x)(2n_+) = \Psi^{\downarrow}(x)(2n^+) = \Psi^{\downarrow}(x)(2n_+^+) = x(n)$$

and,

$$W^{\downarrow}(y)(2n) = \big(-y_v(n) \otimes (-y_h(n))\big) \oplus \big(-y_v(n) \otimes (-y_d(n))\big) \oplus \big(-y_h(n) \otimes (-y_d(n))\big) \oplus 0$$

$$W^{\downarrow}(y)(2n_{+}) = \left(-y_v(n) \otimes y_h(n)\right) \oplus \left(-y_v(n) \otimes y_d(n)\right) \oplus \left(y_h(n) \otimes y_d(n)\right) \oplus 0$$
$$W^{\downarrow}(y)(2n^{+}) = \left(-y_h(n) \otimes y_v(n)\right) \oplus \left(y_v(n) \otimes y_d(n)\right) \oplus \left(-y_h(n) \otimes y_d(n)\right) \oplus 0$$
$$W^{\downarrow}(y)(2n_{+}^{+}) = \left(y_v(n) \otimes y_h(n)\right) \oplus \left(-y_d(n) \otimes y_v(n)\right) \oplus \left(-y_d(n) \otimes y_h(n)\right) \oplus 0$$

also,

$$\begin{aligned} x(2n) &= \Psi^{\downarrow}(x)(2n) \otimes \left(-W^{\downarrow}(y)(2n)\right)x(2n_{+}) \\ &= \Psi^{\downarrow}(x)(2n_{+}) \otimes \left(-W^{\downarrow}(y)(2n_{+})\right)x(2n^{+}) \\ &= \Psi^{\downarrow}(x)(2n^{+}) \otimes \left(-W^{\downarrow}(y)(2n^{+})\right)x(2n_{+}^{+}) \\ &= \Psi^{\downarrow}(x)(2n_{+}^{+}) \otimes \left(-W^{\downarrow}(y)(2n_{+}^{+})\right) \end{aligned}$$

Where $y = (y_v, y_h, y_d)$ and y_v in question is $y_v = W_v(x)$, as well as y_h and y_d. Henceforth, will be prepared and applying the calculation algorithm in pattern recognition applications. The next execution will also conduct comparative efficiency of each formula, the classical Haar wavelet and Haar wavelet max-plus, so that they can give their views on the processing, particularly an image processing.

1.2 Algorithm

In this section will be tested formula of max-plus transformation of the Haar wavelet and Haar wavelet transform classic formula. Means employed to test these formulas is by forming an analytic algorithm of each formula. Of the algorithm is formed, will note the speed of the process, which will appear with a number of steps of the algorithm itself, and the accuracy of information during the calculation process.

1.2.1 The Algorithm

Algorithm 1. Classical Haar wavelet transformation

1. Enter an array of 2^k
2. Count the number D, in which many formed couple $D = 2^{k-1}$
3. Calculate $h_i = \frac{a+b}{2}$, $1 < i < D$
4. Calculate $d_i = d_{i-1} + \frac{a-b}{2}$, $1 < i < D$

5. Calculate $D = \frac{size(h_i)}{2}$
6. During $D \geq 1$, perform steps 3 and 4 again
7. Results of decomposition: h_i (the approximation) and d_i (detail)

From the algorithm analysis process is apparent that there are stages that make the calculation are a fraction. This will make it difficult for the process of synthesis, because the results obtained is not necessarily exactly the same as the initial data. Possible errors that would happen is what can reduce the accuracy of the data, consequently the information on the initial data can also be lost. Furthermore, we will set up an algorithm process the data decomposition analysis using max-plus transformation Haar wavelet.

Algorithm 2. Transformation Haar wavelet max-plus

1. Enter an array of 2^k
2. Calculate $D = 2^{k-1}$
3. Calculate $h_i = \max(a, b)$, $1 < i < D$
4. Calculate $d_i = a \otimes (-b)$, $1 < i < D$
5. Calculate $D = \frac{size(h_i)}{2}$
6. During $D \geq 1$, perform steps 3 and 4 again
7. Results of decomposition: h_i (the approximation) and d_i (detail)

Of the two algorithms above, it appears that the number of steps required to processing is the same. However, the formula for the Haar wavelet max-plus calculation does not appear that resulted in the emergence of fractions. This is of course very useful in the process of decomposition, because it can minimize the risk of lost information due to rounding of numbers. And, of course, can simplify the synthesis process to obtain preliminary data after the analysis process.

1.2.2 The Examples

For comparison, the following example will be able to clarify the difference both in conducting decomposition algorithm for analysis process. Preliminary data:

6	4	2	5	5	6	8	7

Classical Haar wavelet algorithm

6	4	2	5	5	6	8	7

5	$\frac{7}{2}$	$\frac{11}{2}$	$\frac{15}{2}$	1	$-\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

$\frac{17}{2}$	13	$\frac{3}{4}$	-1	1	$-\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

$\frac{43}{4}$	$-\frac{9}{4}$	$\frac{3}{4}$	-1	1	$-\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

Figure 3. Computing the classical Haar wavelet decomposition formula

Max-plus algorithm Haar wavelet

6	4	2	5	5	6	8	7

4	2	5	7	2	-3	-1	1

2	5	2	-2	2	-3	-1	1

2	-3	2	-2	2	-3	-1	1

Figure 4. Decomposition formula for computing the max-plus Haar wavelet

3. CONCLUDING REMARK

From the discussion in the previous section, it appears that the formula for max-plus transformation of the Haar wavelet produces the approximation and the detail of the number that much easier. A result of data analysis on max-plus Haar wavelet is also easier to process synthesis to obtain preliminary data.

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