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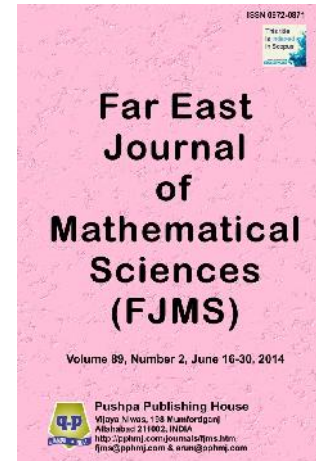
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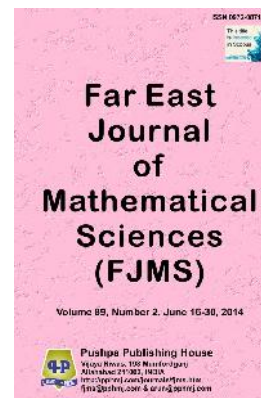
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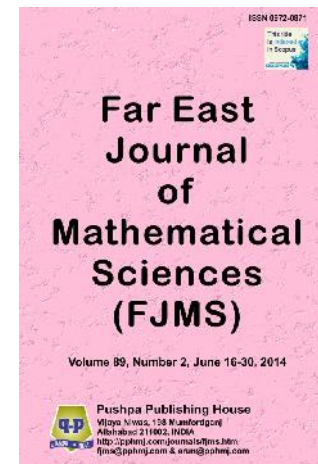
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VISCOELASTIC FLUID FLOW PASS A POROUS CIRCULAR CYLINDER WHEN THE MAGNETIC FIELD INCLUDED

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Abstract

This paper is concerned with the boundary layer flow of steady incompressible and viscoelastic fluid (MHD) passing over horizontal porous circular cylinder. The effect of magnetic field that acts on the fluid is applied and the fluid is assumed to be flowing in a porous

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medium. Governing equations are built from the physical phenomena, i.e., from the continuity equation, the momentum equation and the energy equation. These dimensional governing equations are further reduced by using boundary layer theory and they are converted into non-dimensional form by substituting non-dimensional variables. Further, these non-dimensional equations are transformed into similar equations and then are solved numerically by using finite difference method. Computational results for the flow quantities are presented graphically. We obtain that the temperature profiles increase when both of mixed convection and magnetic parameters increase. The velocity profiles increase when both of mixed convection and magnetic parameters increase.

I. Introduction

The convective of heat transfer, which is a heat transfer from one place to another through the intermediary of a fluid caused by temperature differences, is generally divided into two basic processes, i.e. free convection and forced convection. Free convection is caused by buoyancy forces due to density differences of temperature variation in the fluids (Merkin [14]). In forced convection which is also called as heat advection, fluid moves due to external forces. In addition, when the effect of forces flow in free convection becomes significant, the process then called *mixed convection flow* which is the combination of free and force convection flows. Boundary layer is a narrow region of a thin layer adjacent to the surface of a bluff body when a real fluid flows past the body. The concept of boundary layer flow plays an important role in engineering, automobile, and marine engineering. Due to its importance in many engineering applications, the mixed convection boundary layer flow of non-Newtonian fluid in the presence of magnetic field has been investigated by many researchers, they are Hsiao [9], Ahmed et al. [1], Ghosh and Shit [8], Aurangzaib et al. [4], etc. These studies are also used for modeling and simulation.

Ghosh and Shit [8] numerically investigated about boundary layer flow of viscoelastic fluid flow with short memory (obeying Walters' B fluid model) passing over a hot vertical porous plate with the presence of magnetic

field. The result of their research showed that Prandtl number has more pronouncing effect on the temperature distribution rather than the viscosity parameter as well as the thermal radiation parameter. Anwar et al. [2] observed the boundary layer flow in circular cylinder using Keller Box method as its numerical solution method.

In order to study the mixed convection boundary layer flow of non-Newtonian fluid in the presence of magnetic field, we consider a research on viscoelastic fluids of steady incompressible fluid which is past a horizontal porous circular cylinder. The geometry illustration and the coordinate system for the problem are shown in Figure 1. It is assumed that the uniform velocity of ambient fluid is $\frac{1}{2}U_\infty$ and temperature of ambient fluid is T_∞ .

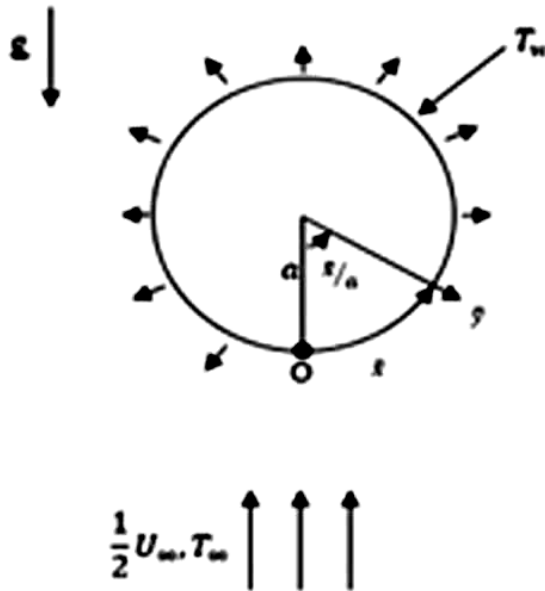


Figure 1. Coordinate system of the problem.

II. Mathematical Formulation of the Problem

Considering the physical model and coordinate system of this problem, we can formulate mathematical model as follows.

A. Governing equation

Due to the physical phenomena, when the fluid past the horizontal porous circular cylinder, volume force, Boussinesq approximation, conservation law, and boundary layer theory, continuity, momentum, and energy equations of the system can be written as follows:

Continuity equation

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0. \quad (1)$$

Momentum equation

$$\begin{aligned} \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = & \bar{u}_e \frac{\partial \bar{u}_e}{\partial \bar{x}} + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \left(\frac{1}{\rho} \sigma B_0^2 + \frac{\nu}{\kappa} \right) (\bar{u} - \bar{u}_e) \\ & + g\beta(T - T_\infty) \sin\left(\frac{x}{a}\right) - \frac{k_0}{\rho} \left[\bar{u} \left(\frac{\partial^3 \bar{u}}{\partial \bar{x} \partial \bar{y}^2} \right) \right. \\ & \left. + \bar{v} \frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - \frac{\partial \bar{u}}{\partial \bar{y}} \left(\frac{\partial^2 \bar{u}}{\partial \bar{y} \partial \bar{x}} \right) + \frac{\partial \bar{u}}{\partial \bar{x}} \left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \right) \right]. \end{aligned} \quad (2)$$

Energy equation

$$\left(\bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} \right) = \alpha \frac{\partial^2 T}{\partial \bar{y}^2} \quad (3)$$

subjected to boundary condition

$$\bar{u} = \bar{v} = 0, T = T_w, \text{ at } \bar{y} = 0,$$

$$\bar{u} = \bar{u}_e, \frac{\partial \bar{u}}{\partial \bar{y}} = 0, T = T_\infty, \text{ when } y \rightarrow \infty, \quad (4)$$

where \bar{x} and \bar{y} are Cartesian coordinates measured along the surface of the cylinder starting from the lower stagnation point of the cylinder and \bar{y} is the coordinate measured normal to the surface of the cylinder, \bar{u} and \bar{v} are the velocity components, \bar{u}_e is the velocity outside the boundary layer, T_w is

constant cylinder wall temperature, T is the fluid temperature, g is the acceleration due to gravity, k_0 is the viscoelasticity, κ is the permeability, B_0 is magnetic field, σ is electrical conductivity, ρ is density of the fluid, and ν , α , β are kinematic viscosity, thermal diffusivity, and thermal expansion coefficient, respectively.

These equations are called *dimensional boundary layer equation*. These dimensional governing equations further transform into non-dimensional governing equations by substituting non-dimensional variables. The non-dimensional variables are defined as (Anwar et al. [2]):

$$\begin{aligned} x &= \frac{\bar{x}}{a}, \quad y = \text{Re}^{\frac{1}{2}} \left(\frac{\bar{y}}{a} \right), \quad u = \frac{\bar{u}}{U_\infty}, \\ v &= \text{Re}^{\frac{1}{2}} \left(\frac{\bar{v}}{U_\infty} \right), \quad u_e = \frac{\bar{u}_e(\bar{x})}{U_\infty}, \quad \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \end{aligned} \quad (5)$$

where $\text{Re} = \frac{U_\infty a}{\nu}$ is Reynolds number. By substituting equation (5) into dimensional governing equations (1), (2) and (3), we obtain the following non-dimensional equations, i.e.:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (6)$$

Momentum equation

$$\begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= u_e \frac{du_e}{dx} + \frac{\partial^2 u}{\partial y^2} - (M + \phi)(u - u_e) + \lambda \theta \sin x \\ &- K \left[\frac{\partial}{\partial x} \left(u \frac{\partial^2 u}{\partial y^2} \right) + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} \right]. \end{aligned} \quad (7)$$

Energy equation

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2}. \quad (8)$$

The boundary conditions (4) become:

$$u = v = 0, \theta = 1, \text{ at } y = 0, \quad u = u_e(x), \frac{\partial u}{\partial y} = 0, \theta = 0, \text{ when } y \rightarrow \infty, \quad (9)$$

where K , M , ϕ , λ and Pr are dimensionless parameters, which are defined as follows:

$$K = \frac{(k_0 U_\infty)}{(\rho \nu)} \quad (\text{viscoelastic parameter}),$$

$$M = \frac{\sigma B_0^2 a}{\rho U_\infty} \quad (\text{magnetic parameter}),$$

$$\phi = \frac{\nu a}{\kappa U_\infty} \quad (\text{porosity parameter}),$$

$$\lambda = \frac{Gr}{\text{Re}^2} \quad (\text{mixed convection parameter}),$$

$$Gr = \frac{g \beta (T_w - T_\infty) a^3}{\nu^2} \quad (\text{Grashof number}),$$

$$\text{Pr} = \frac{\nu}{\alpha} \quad (\text{Prandtl number}).$$

In this research, we use $\lambda > 0$, which should be mentioned that this condition corresponds to assisting flow (heated cylinder).

B. Solution of the problem

Let us introduce the similarity variable to solve the set of non-dimensional governing equations (6) to (8) and boundary conditions (9), following (Anwar et al. [2]):

$$\begin{aligned} \psi &= x f(x, y), \\ \theta &= \theta(x, y), \end{aligned} \quad (10)$$

where ψ is stream function which is defined as follows:

$$u = \frac{\partial \Psi}{\partial y} \text{ and } v = \frac{\partial \Psi}{\partial x} \tag{11}$$

with the use of above variables into equations (6) to (8) and substitute $u_e = \sin x$ (Merkin [14]), we obtain:

Momentum equation

$$\begin{aligned} & \frac{\partial^3 f}{\partial y^3} + f \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y} \right)^2 + \frac{\sin x \cos x}{x} - (M + \phi) \left(\frac{\partial f}{\partial y} - \frac{\sin x}{x} \right) \\ & + \lambda \frac{\sin x}{x} \theta - K \left(2 \frac{\partial f}{\partial y} \frac{\partial^3 f}{\partial y^3} - f \frac{\partial^4 f}{\partial y^4} - \left(\frac{\partial^2 f}{\partial y^2} \right)^2 \right) \\ & - Kx \left(\frac{\partial^2 f}{\partial x \partial y} \frac{\partial^3 f}{\partial y^3} - \frac{\partial f}{\partial x} \frac{\partial^4 f}{\partial y^4} + \frac{\partial f}{\partial y} \frac{\partial^4 f}{\partial x \partial y^3} - \frac{\partial^2 f}{\partial y^2} \frac{\partial^3 f}{\partial x \partial y^2} \right) \\ & = x \left(\frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2} \right). \end{aligned} \tag{12}$$

Energy equation

$$\frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + f \frac{\partial \theta}{\partial y} = x \left(\frac{\partial f}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x} \frac{\partial \theta}{\partial y} \right) \tag{13}$$

and subjected to the boundary conditions (9), which become:

$$\begin{aligned} f = \frac{\partial f}{\partial y} = 0, \theta = 1, \text{ at } y = 0, \\ \frac{\partial f}{\partial y} = \frac{\sin x}{x}, \frac{\partial^2 f}{\partial y^2} = 0, \theta = 0 \text{ while } y \rightarrow \infty. \end{aligned} \tag{14}$$

At the lower stagnation point ($x \approx 0$), both equations (12) and (13) are reduced into highly non-linear ordinary differential equations

$$f''' + ff'' - f'^2 + 1 - (M + \phi)(f' - 1) + \lambda\theta - K(2f'f''' - ff^{(4)} - f''^2) = 0, \quad (15)$$

$$\frac{1}{Pr}\theta'' + f\theta' = 0 \quad (16)$$

subjected to boundary conditions

$$\begin{aligned} f(0) = f'(0) = 0, \theta(0) = 1, \\ f'(\infty) = 1, f''(\infty) = 0, \theta(\infty) = 0, \end{aligned} \quad (17)$$

where ' denotes the differentiation with respect to y .

The physical quantities of principal interest for this problem are heat transfer coefficient (Q_w). The dimensionless heat transfer coefficient Q_w (Anwar et al. [2]) is given as

$$Q_w = -\theta'(x, 0). \quad (18)$$

III. Result and Discussion

The systems of equations (15)-(17) are solved numerically for some values of the mixed convection parameter and magnetic parameter using second order finite difference method.

The comparison of the present result for the heat transfer coefficient Q_w with existing works done by Anwar et al. [2] has been made at $M = 0$, $\phi = 0$, $Pr = 1$, and $\lambda = 0$ and 1. The result shows that the heat transfer coefficient decreases when the viscoelastic parameter (K) increases. The details of the comparison values can be seen in Table 1.

In this study, the mixed convective boundary layer flow on non-Newtonian fluid in the presence of magnetic field past a porous circular cylinder is investigated numerically using FDM scheme. The fluid is viscoelastic fluid, which has characteristics both of viscous and elastic, with the presence of magnetic field. The objective of the present analysis is to

study the temperature profiles and velocity profiles of viscoelastic fluid flow with the variation of mixed convection parameter (λ) and magnetic parameter (M).

Table 1. Comparison values of heat transfer coefficient $-\theta'(0)$ for the various values of K when $\lambda = 0$ and $\lambda = 1$ at $Pr = 1, M = 0, \phi = 0$

K	$\lambda = 0$		$\lambda = 1$	
	Anwar et al. [2]	Present Result	Anwar et al. [2]	Present Result
0.01	0.56913	0.5691	0.613861	0.6138
0.1	0.558175	0.5560	0.600089	0.6007
0.2	0.548077	0.5484	0.587800	0.5859
0.4	0.532036	0.5345	0.568851	0.5672
0.6	0.519487	0.5171	0.554389	0.5525
0.7	0.514101	0.5123	0.548256	0.5484
0.9	0.504626	0.5037	0.537559	0.5383
1	0.500411	0.5000	0.532833	0.5330

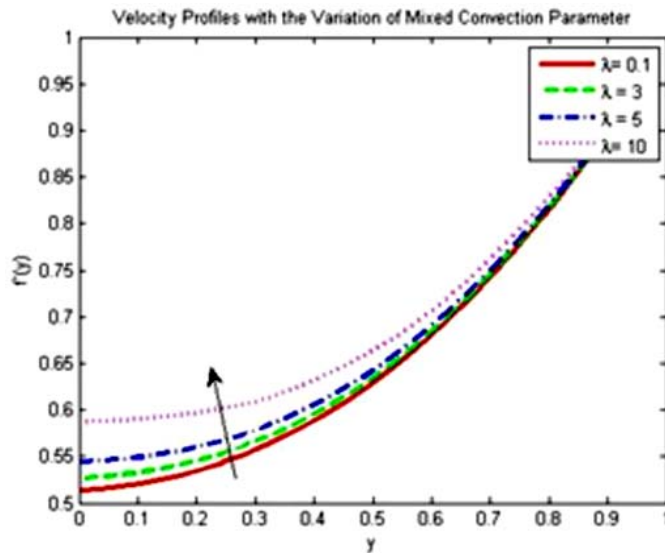


Figure 2. Velocity profiles with fixed $M = 0.5, Pr = 1, K = 1, \phi = 0.1$ and variation of mixed convection parameter (λ).

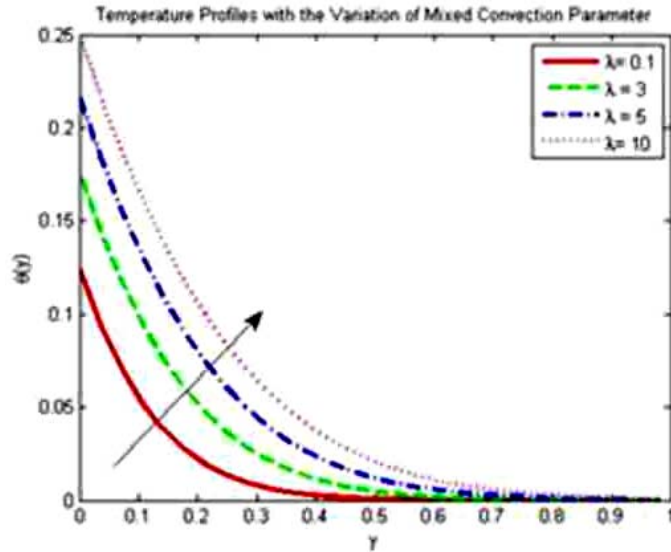


Figure 3. Temperature profiles with fixed $M = 0.5$, $Pr = 1$, $K = 1$, $\phi = 0.1$ and variation of mixed convection parameter (λ).

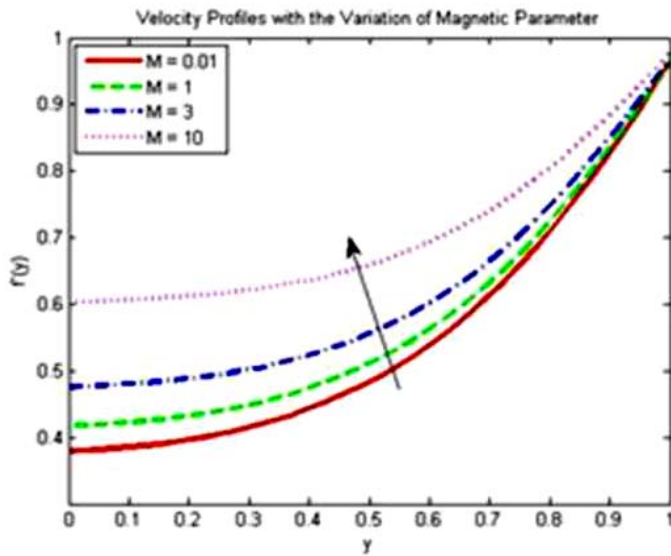


Figure 4. Velocity profiles with fixed $\lambda = 2$, $Pr = 1$, $K = 1$, $\phi = 0.1$ and variation of magnetic parameter (M).

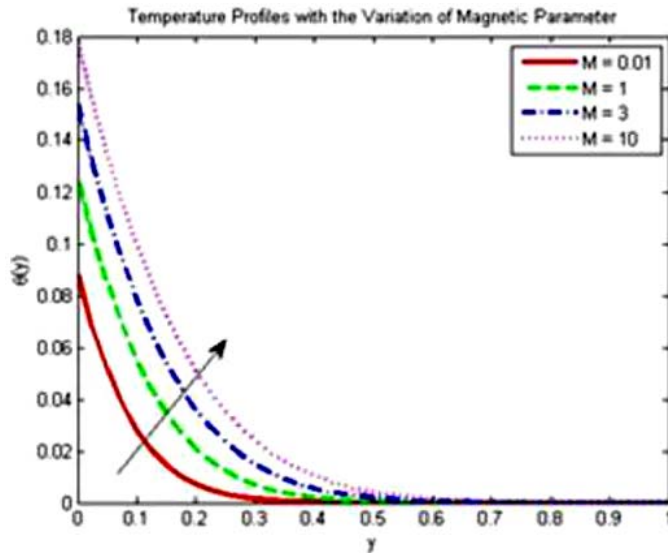


Figure 5. Temperature profiles with fixed $\lambda = 2$, $Pr = 1$, $K = 1$, $\phi = 0.1$ and variation of magnetic parameter (M).

The influence of mixed convection parameter on the velocity profiles and temperature profiles can be seen in Figure 2 and Figure 3, respectively. These numerical results have been made at fixed values of $M = 0.5$, $\phi = 0.1$, $K = 1$ and $Pr = 1$. The velocity profiles in Figure 2 and temperature profiles in Figure 3 increase when mixed convection parameter increases. The temperature distribution in this problem increases when cylinder is heated ($\lambda > 0$). Then the velocity profiles increase because the density of the fluid decreases as long as the temperature of the fluid increases. In this condition, the buoyancy force that occurs on the fluid makes the fluid move faster than in normal condition (without mixed convection influence).

Figure 4 and Figure 5 show the effect of the increasing value of magnetic parameter (M) on the physical properties of the fluid. These numerical results have been made at fixed values of $Pr = 1$, $\lambda = 2$, $K = 1$, $\phi = 0.1$. The result shows that the velocity and temperature profiles increase when magnetic parameter on the fluid increases. This is expected because the increasing of magnetic parameter will affect the Lorentz force. This

condition also shows that the high temperature of the fluid will affect the velocity of the fluid.

The numerical result also shows that temperature profiles decrease when the Prandtl number increases. However, the velocity profiles decrease when viscoelastic parameter, porosity parameter of the cylinder, and Prandtl number increase.

IV. Conclusions

Viscoelastic fluid flow with the presence of magnetic field past a porous circular cylinder is investigated numerically by using finite difference method. The effects of the viscoelastic parameter, mixed convection parameter, magnetic parameter, porosity parameter, and Prandtl number on the physical properties of the fluid have been examined. The results show that the velocity profiles decrease when the viscoelastic parameter, porosity parameter, and Prandtl number increase. Velocity profiles increase when both of mixed convection parameter and magnetic field parameter increase. Temperature profiles increase when viscoelastic parameter, mixed convection parameter, magnetic parameter increase. Temperature profiles decrease when the Prandtl number increases.

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